ETDC Indonesian Journal of Research and Educational Review
Volume 5, No. 4, 2026, pp 2565 - 2581
e–ISSN 2809-3682
p–ISSN 2809-7505
https://doi.org/10.51574/ijrer.v5i4.5471

Students' Mathematical Representation in PISA Space and Shape Tasks Across Gregorc's Thinking Styles

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Article Info

Article history:

Received May 08, 2026
Accepted July 23, 2026
Published July 27, 2026

Keywords:

Differentiated Instruction; Gregorc Thinking Style; Mathematical Representation; PISA Space and Shape; Qualitative Case Study.

ABSTRACT

This qualitative descriptive study investigates high school students' mathematical representation ability in solving PISA Space and Shape problems, analyzed through Gregorc's thinking style model. Eight tenth-grade students at Islamic High Secondary School DDI Takkalasi (2025/2026 academic year) were purposively selected based on a Gregorc Thinking Style Questionnaire, representing two students per style: Concrete Sequential (CS), Abstract Sequential (AS), Concrete Random (CR), and Abstract Random (AR). Data gathered through questionnaires, a six-item mathematical representation test adapted from PISA released items, and semi-structured interviews were analyzed using Miles, Huberman, and Saldaña's framework, with credibility ensured via source triangulation. Results demonstrate distinct representation profiles across styles: CS students systematically integrated verbal, visual, and symbolic representations; AS students prioritized conceptual understanding by linking visual confirmation to symbolic formalization through verbal justification; CR students utilized visual representation for exploration but struggled with accurate symbolic translation; and AR students relied on mental visualization and intuition, leaving key reasoning unverified. These findings indicate that thinking styles systematically influence mathematical representation behaviors and suggest that evaluating representation ability solely through written work risks underestimating conceptual understanding, particularly in random-oriented thinkers. The study provides critical insights for developing differentiated instructional strategies that align mathematical task design with students' dominant cognitive processing styles.

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1. INTRODUCTION

Mathematics education plays a pivotal and strategic role in fostering logical, analytical, and creative thinking competencies essential for addressing complex global challenges in the twenty-first century (Mierluş-Mazilu, I., & Yilmaz, 2023; Szabo et al., 2020). Internationally, the Program for International Student Assessment (PISA) serves as a benchmark for evaluating student performance, emphasizing the practical application of mathematical literacy in real-world contexts over mechanical procedural memorization (Almarashdi & Jarrah, 2023; Incikabi, 2026). A fundamental component of mathematical literacy is mathematical representation ability, which the National Council of Teachers of Mathematics (Niss & Jablonka, 2020) identifies as a core process standard crucial for deep conceptual understanding (Nanda & Rani, 2025).

Theoretical perspectives define representation as any visible structure or symbol—such as diagrams, graphs, concrete objects, algebraic notation, or spreadsheet displays—that stands for an underlying mathematical idea or relationship (Lutfi & Juandi, 2023; Ratumanan et al., 2022). Serving as a critical cognitive bridge between abstract conceptual understanding and operational problem-solving, effective representation enables learners to systematically organize complex information and formulate efficient solution strategies (Liang & She, 2023; Tajudin & Chinnappan, 2016).

Within the PISA assessment framework, the Space and Shape content domain relies heavily on spatial reasoning, geometric visualization, and the conceptual manipulation of two- and three-dimensional objects (Nuryadi et al., 2024). Problem-solving success in this domain is inherently tied to a student's ability to construct appropriate representations—moving fluidly from situational context interpretation to the selection of optimal representational modalities (Nilimaa, 2023; Santos-Trigo, 2024). Consequently, representation is recognized not merely as an auxiliary technical skill but as the core engine of spatial reasoning and geometric problem-solving.

Despite its critical importance, empirical evidence consistently demonstrates that Indonesian students exhibit unsatisfactory mathematical representation abilities (Al Addawiyah & Basuki, 2022; Lutfi & Dasari, 2023; Lutfi & Juandi, 2023). PISA 2022 results highlight a significant national performance deficit: Indonesia's mean mathematics score remains substantially below the OECD average, with only 18% of students attaining Level 2 proficiency compared to the OECD benchmark of 69% and over 85% in top-performing Asian systems (Latifah & Kismiantini, 2025). Most Indonesian learners fail to master basic mathematical competencies, and high-level proficiency (Levels 5–6) remains virtually unreached.

Recent educational literature indicates that students' representational processes are significantly modulated by cognitive characteristics, specifically individual thinking styles (Khatin-Zadeh, 2022; Sobirin et al., 2023). Grounded in Gregorc's model, thinking style is defined along two primary dimensions: perception (concrete vs. abstract) and ordering (sequential vs. random) (Pramono et al., 2025; Santoso et al., 2025). The resulting taxonomy yields four distinct cognitive profiles—Concrete Sequential (CS), Abstract Sequential (AS), Concrete Random (CR), and Abstract Random (AR)—each hypothesized to uniquely influence how students perceive problems, construct representations, and execute solution pathways.

Practical classroom observations at Islamic High Secondary School DDI Takkalasi reveal widespread student difficulty in translating verbal geometric problems into appropriate visual models or symbolic representations—a challenge strongly supported by existing literature (Castro et al., 2022; Mulyaningsih et al., 2020; Rosyidin & Rosyidi, 2022). While prior studies have addressed geometric visualization

(Mulyaningsih et al., 2020), problem-solving strategies (Fauzi et al., 2020), or representational transitions (Castro et al., 2022) in isolation, a noticeable research gap exists regarding the intersection of Gregorc thinking styles and mathematical representation within PISA Space and Shape tasks, particularly in Islamic senior high school (MA) contexts.

Addressing this literature gap offers crucial theoretical and practical insights into how distinct cognitive styles influence information processing and strategic representation choices, thereby informing adaptive, differentiated instructional design. Accordingly, this study analyzes the mathematical representation ability of tenth-grade students at Islamic High Secondary School DDI Takkalasi when solving PISA Space and Shape problems across four specific Gregorc thinking styles: (1) Concrete Sequential, (2) Abstract Sequential, (3) Concrete Random, and (4) Abstract Random.

2. METHOD

This study employed a qualitative approach with a descriptive design intended to analyze students' mathematical representation ability in solving PISA Space and Shape problems in relation to thinking style. A qualitative approach was chosen because the study sought an in-depth understanding of how students with different thinking styles build, use, and connect various forms of mathematical representation during problem solving.

The study was conducted during the even semester of the 2025/2026 academic year, at Islamic High Secondary School DDI Takkalasi, South Sulawesi Province, Indonesia. Participants were tenth-grade students at Islamic High Secondary School DDI Takkalasi in the 2025/2026 academic year, selected purposively on the grounds that tenth-grade students (1) had developed sufficient analytical thinking to solve mathematical problems, particularly those involving Space and Shape content; (2) had studied relevant geometry material and therefore possessed the conceptual foundation needed to understand and solve PISA-oriented Space and Shape items; and (3) possessed adequate communicative ability, facilitating data collection on their thinking processes and representation ability through interviews.

Subject selection proceeded through the following steps: (1) all 101 tenth-grade students were considered as prospective participants; (2) a mathematics ability test was administered to identify students' baseline mathematical competence; (3) twenty students (approximately 20% of the 101) who scored 75 or above were shortlisted as prospective respondents, to secure a pool with relatively comparable mathematical ability; (4) the shortlisted students completed a thinking-style questionnaire; (5) questionnaire results were analyzed and students were grouped into four thinking-style categories; (6) the two highest-scoring students within each category were selected, yielding two students per style; and (7) eight students judged to best represent the characteristics of their respective thinking styles were confirmed as the final participants (Table 1).

Table 1. Participant Selection Results

No.	Initials	Thinking-Style Score	Thinking-Style Category	Subject Code
1	HSF	44	Concrete Sequential (CS)	CS-1
2	DSR	44	Concrete Sequential (CS)	CS-2
3	WDG	48	Abstract Sequential (AS)	AS-3
4	ZLF	40	Abstract Sequential (AS)	AS-4
5	AQR	40	Concrete Random (CR)	CR-5
6	ASW	36	Concrete Random (CR)	CR-6
7	APR	44	Abstract Random (AR)	AR-7
8	ATB	44	Abstract Random (AR)	AR-8

Four techniques were used: (1) a mathematics ability test, administered to all tenth-grade students to identify those with relatively equal baseline competence, minimizing the confound of prior mathematical achievement in the subsequent representation analysis; (2) a Gregorc thinking-style questionnaire, administered to the mathematically comparable subgroup to classify them into the four thinking styles and to select the eight target participants; (3) a mathematical representation test comprising six essay items adapted from PISA Space and Shape, targeting visual, symbolic, and verbal representation in spatial reasoning and problem solving; and (4) semi-structured interviews conducted after the test, individually, audio-recorded and supported by field notes, to probe participants' reasoning, strategies, and representational choices in greater depth.

The primary instrument was the researcher herself (human instrument), responsible for planning, collecting, analyzing, and interpreting the data. Supporting instruments comprised: (1) a mathematics ability test used to screen participants with comparable baseline competence; (2) a thinking-style questionnaire, selected for its established validity and reliability in classifying students' concrete/abstract and sequential/random information-processing tendencies; (3) a mathematical representation test consisting of six essay items adapted from the 2003 and 2012 PISA Space and Shape released items, designed to elicit visual, symbolic, and verbal representations; and (4) a semi-structured interview guide addressing participants' problem comprehension, representational choices and justifications, difficulties encountered, and the relation between thinking style and solution strategy.

Data were analyzed following the qualitative analysis model of Miles, Huberman, and Saldaña, comprising data condensation, data display, and conclusion drawing/verification. Data condensation involved selecting, focusing, simplifying, abstracting, and transforming data obtained from field notes, test results, interview transcripts, and supporting documents—including grouping subjects by Gregorc thinking style, identifying the forms of representation (visual, verbal, symbolic) evident in their work, and selecting data that illuminated representational characteristics relevant to the Space and Shape items. Data display involved systematically presenting condensed data as student work samples, interview transcript summaries, and analytical narratives, comparing test responses with interview accounts item by item, and subsequently synthesizing findings across the two subjects within each thinking-style category. Conclusion drawing and verification involved identifying patterns and tendencies in visual, symbolic, and verbal representation use within and across thinking-style categories and verifying these conclusions through source triangulation.

Credibility was established through source triangulation, comparing and cross-checking consistency among data obtained from the four collection techniques (mathematics ability test, thinking-style questionnaire, representation test, and interview). Where test and interview data converged, the resulting characterization was treated as credible; findings were additionally corroborated with information obtained from the participants' mathematics teacher.

3. RESULTS AND DISCUSSION

Results

Overview

Analysis proceeded for each of the four Gregorc thinking-style groups (CS, AS, CR, AR) across the six mathematical representation test items, adapted from PISA Space and Shape released items: (1) triangle construction from verbal conditions, (2) garage roof area (requiring the Pythagorean theorem), (3) Ferris-wheel height above a riverbed, (4) kite-string length at a 45° angle, (5) selection of garden-fence designs satisfying a fixed perimeter, and (6) identification of dice nets satisfying an opposite-faces-sum-to-seven rule. For each item, students' visual (V), symbolic (S), and verbal (Vb) representations were rated qualitatively as Good, Adequate, Partial, Poor, or Absent/Not produced, based on triangulated written-response and interview data. Item 2 is used as a running illustrative example across the four groups (Figures 1–4) because it required students to coordinate all three representation forms to reach a correct answer, making group differences particularly visible.

Concrete Sequential (CS)

Both CS subjects displayed a consistent, integrated three-representation pattern across the six items, typically opening with a verbal restatement of the given information, followed by a visual model, a symbolic procedure, and a closing verbal conclusion (Table 2).

Table 2. Representation Quality of Concrete Sequential (CS) Subjects

Item	CS-1: Visual / Symbolic / Verbal	CS-2: Visual / Symbolic / Verbal
Triangle construction	Adequate / – / Good	Adequate / – / Good
Roof area	Good / Good / Good	– / Poor (wrong dimension) / Good
Ferris-wheel height	Good (reused given diagram) / Good / Good	– (reused given diagram) / Good / Good (most elaborate)
Kite-string length	Good / Good / Good	Good (creative square–diagonal model) / Good (unsimplified) / Good
Garden-fence design	Good / Good (verbal used for one design) / Good	Good / Not documented (verified mentally) / Good
Dice net	Good / Good / Good	Absent (mental) / Absent (mental) / Adequate (general statement)

On item 2, which asked students to determine the total roof area of a garage using the Pythagorean theorem, CS-1 produced a complete and accurate sequence: a labelled cross-sectional sketch of the roof, a step-by-step Pythagorean derivation of the missing slant side, and a concluding statement of the total area (31.80 m²), illustrating the integrated verbal–visual–symbolic pattern typical of this group (Figure 1). CS-2 solved the same item using a structured "Given–Asked–Solution" format but substituted the horizontal half-width of the roof (2.50 m) directly into the area formula without first applying the Pythagorean theorem to obtain the true slant length; the interview revealed that CS-2 understood conceptually that the slant side should have been computed first, indicating that the error stemmed from incomplete written documentation rather than a conceptual gap.

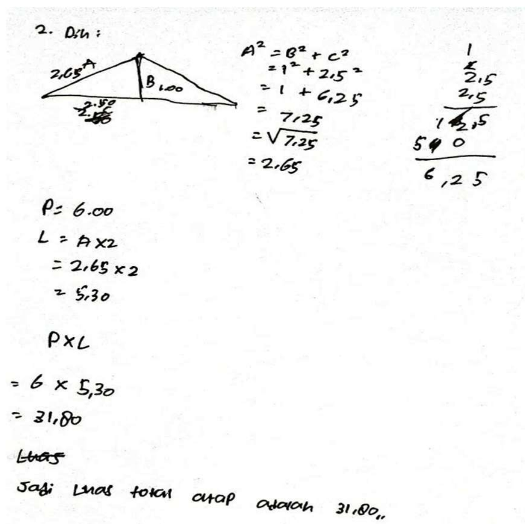

Figure 1. CS-1's Response to Item 2 (Roof-Area Problem)

Across the six items, verbal representation consistently opened and closed each solution: both subjects recorded the given and required information before turning to visual or symbolic work, a habit both explained to avoid re-reading the problem repeatedly and to reduce errors. Visual representation was used flexibly: new diagrams were constructed on items 1, 2, 4, 5, and 6, while the diagram supplied with item 3 was judged sufficient and reused directly. Symbolic representation varied in completeness—CS-1 documented every procedural step, including the multi-stage Pythagorean derivation on item 2, whereas CS-2 sometimes verified answers mentally (items 5 and 6) without recording the calculation, despite being able to reconstruct and justify the reasoning fully in interviews. Overall, the CS profile is characterized by a stable, linear cycle—verbal organization of information, visual modelling of the geometric situation, symbolic execution or verification, and verbal communication of the conclusion—with individual differences between CS-1 and CS-2 lying mainly in the completeness of written documentation rather than in the underlying problem-solving approach.

Abstract Sequential (AS)

AS subjects favored a deductive sequence in which a general rule or concept was articulated before any numerical substitution, with visual representation functioning mainly to confirm rather than generate understanding (Table 3).

Table 3. Representation Quality of Abstract Sequential (AS) Subjects

Item	AS-3: Visual / Symbolic / Verbal	AS-4: Visual / Symbolic / Verbal
Triangle construction	Adequate (two positional inaccuracies) / – / –	Good / – / –
Roof area	– (used given diagram) / Good / Good	– / Adequate (rounding error) / Adequate
Ferris-wheel height	– (used given diagram) / Good (derived implicit 10 m) / Good	– / Good / Good
Kite-string length	Good / Good (exact form 150√2) / Good	Good / Good (exact form) / Limited
Garden-fence design	– / Partial (verified design D only) / Adequate (unverified elimination)	– / Partial (verified D only; A, C wrongly eliminated) / Adequate
Dice net	Good (redrawn nets with verification marks) / Implicit / Good	Good (redrawn nets, complete) / Good (all six pairs verified) / Good

On item 2, both AS subjects favored writing a general formula before substituting numbers: AS-4 wrote L = 2 × (P × l) before substitution, explaining in interview that stating the rule first clarified why the area had to be doubled (Figure 2). This "concept-before-calculation" pattern recurred on item 3, where AS-4 wrote "Height of point M = radius + 10" as a verbal–symbolic hybrid rule before executing the arithmetic. AS-3 showed a parallel but more explanatory pattern, embedding a written justification for each procedural step—a habit the subject described as necessary so as not to lose track of the reasoning on multi-step problems.

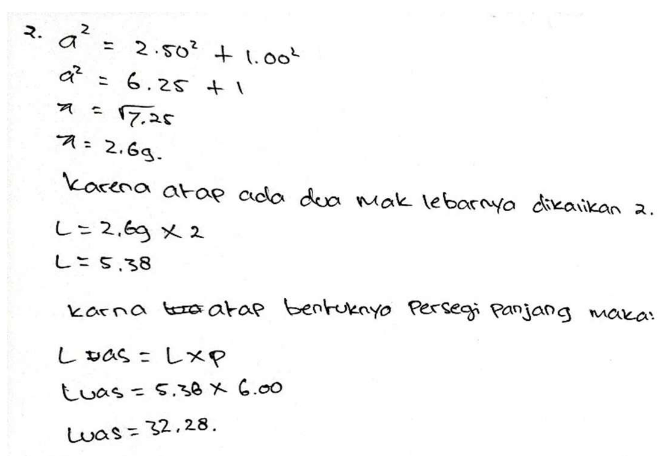

Figure 2. AS-3's Response to Item 2

Visual representation for both AS subjects functioned mainly as a means of confirming an already-formed conceptual understanding rather than generating new information: AS-3 produced a small exploratory sketch before a more carefully finished diagram on item 1, while AS-4 relied on mental visualization for most items and drew only when a new spatial relationship needed to be checked, as on item 6. Symbolic representation reflected strong conceptual orientation, including a preference for exact irrational forms (150√2 m) over decimal approximations, justified explicitly as preserving precision. However, on item 5 both subjects generalized their elimination of alternative garden-fence designs from visual impression rather than complete numerical verification, resulting in an incomplete answer; both acknowledged in interviews that they had not checked every design individually. Overall, the AS profile is marked by representation used to build and safeguard conceptual understanding: verbal representation articulates and justifies the reasoning, visual representation confirms it, and symbolic representation formalizes it only once the underlying idea is considered secure.

Concrete Random (CR)

CR students' problem solving was visibly anchored in visual representation, constructed first and most elaborately in almost every item, but not always translated successfully into symbolic procedure (Table 4).

Table 4. Representation Quality of Concrete Random (CR) Subjects

Item	CR-5: Visual / Symbolic / Verbal	CR-6: Visual / Symbolic / Verbal
Triangle construction	Good (two-stage: exploratory sketch, then final drawing) / – / –	Good (tick-mark convention for equal segments) / – / –
Roof area	Adequate (used half-width instead of slant side) / Adequate (procedure correct, value wrong) / Partial	Good (rich 3-D sketch) / Poor (Pythagorean step left incomplete) / Adequate
Ferris-wheel height	Good (most complete diagram of the set) / Good / Good	Good / Good / Good (deliberate written justification)
Kite-string length	Good / Absent (guessed "about 200 m") / Adequate (directional estimate)	Good (contextual ship-and-kite sketch) / Absent (lacked 45°-45°-90° knowledge) / Adequate (estimate)
Garden-fence design	Poor (not used as calculation basis) / Absent / Poor (invalid reasoning)	Absent / Implicit, inaccurate / Poor (false conclusion)
Dice net	Adequate (only one net checked) / Absent (mental) / Partial	Good (physical paper cube model) / Absent (written) / Good (detailed, reflective)

On item 2, CR-5 correctly recognized the triangular cross-section of the roof but substituted the horizontal half-width (2.50 m) for the slant side without applying the Pythagorean theorem, while CR-6 applied the theorem correctly up to $1^2 + 2.5^2 = 7.25$ but did not take the square root, using 7.25 directly as a length (Figure 3); interview probing revealed that both subjects understood the missing step once prompted to reflect, suggesting the errors were procedural lapses rather than conceptual gaps.

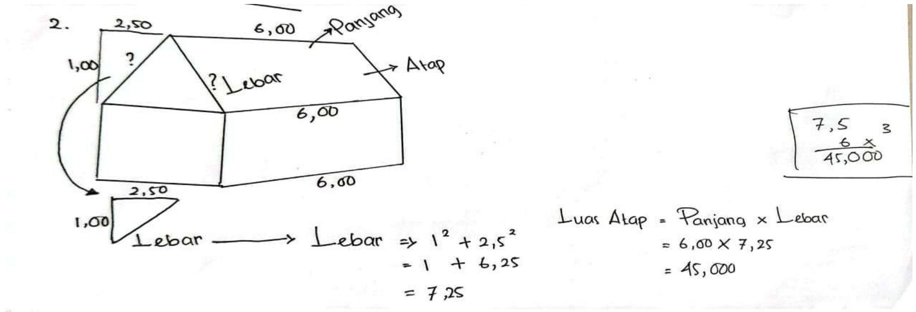

Figure 3. CR-6's Response to Item 2

CR-6, in particular, extended visual representation into physically manipulable models: on the dice-net item, the subject cut and folded a paper cube from scrap paper to verify which nets satisfied the "opposite faces sum to seven" rule, then generalized the resulting face-pairing pattern to check the remaining nets without repeating the physical folding—an efficient strategy reflecting a concrete, hands-on approach to an inherently three-dimensional problem. On item 4, both subjects produced an accurate visual model of the situation yet could not proceed symbolically because neither recalled the 45°-45°-90° triangle relationship, resulting in verbal estimates ("about 200 m," "more than 150 m") rather than calculated answers. Item 5 proved weakest for both subjects, whose design choices rested on visual impression without numerical verification of perimeter, leading to incomplete or incorrect conclusions. Overall, the CR profile shows visual representation functioning as the primary tool for exploring and understanding a problem, but the transformation of visual insight into a correct symbolic procedure remained inconsistent, succeeding mainly when the constructed diagram already contained all the information needed for calculation, as on item 3.

Abstract Random (AR)

AR students used representation selectively, producing written work mainly when judged necessary and relying heavily on mental visualization and intuitive judgment for the remainder (Table 5).

Table 5. Representation Quality of Abstract Random (AR) Subjects

Item	AR-7: Visual / Symbolic / Verbal	AR-8: Visual / Symbolic / Verbal
Triangle construction	Poor (several conditions unmet) / – / –	Poor (built without reference to stated conditions) / – / –
Roof area	Good / Adequate (area not doubled for both roof faces) / Absent	Absent (relied on given diagram) / Poor (wrong dimension) / Absent
Ferris-wheel height	Absent (relied on given diagram) / Adequate (correct value; writing slip "140/20 = 70") / Absent	Absent / Good (correct reasoning; ambiguous notation) / Good
Kite-string length	Good / Good ($\sqrt{2}\approx1.4$ approximation → 210 m) / Absent	Absent (mental) / Good (exact form $150\sqrt{2}$) / Good
Garden-fence design	Absent ("Design D" with no supporting work) / Absent / Absent	Absent / Adequate (verified A, C, D) / Adequate

Item	AR-7: Visual / Symbolic / Verbal	AR-8: Visual / Symbolic / Verbal
Dice net	Absent / Absent / Poor (incorrect: nets III & IV)	Partial (nets marked, no final answer recorded) / Absent / Absent

On item 2, both subjects showed a partial but incomplete symbolic procedure: AR-7 correctly applied the Pythagorean theorem but multiplied only one roof face's area by the length of the garage, omitting the final doubling step for the second, symmetric face (Figure 4), while AR-8 substituted an incorrect dimension read directly from the given diagram. Neither subject accompanied their symbolic work with verbal justification, and in both cases the researcher's follow-up questioning during interview elicited an accurate account of the missing step, indicating that the underlying understanding existed but had not been documented.

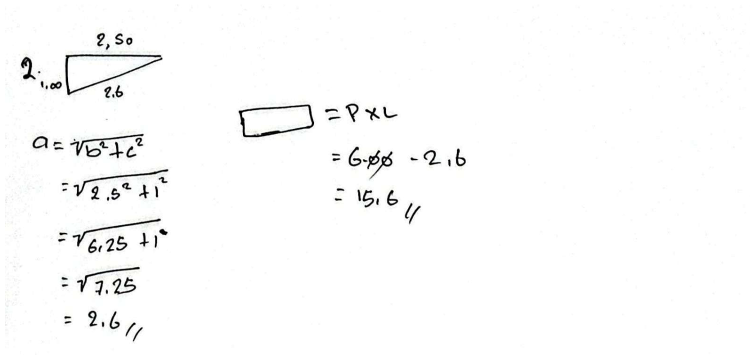

Figure 4. AR-7's Response to Item 2

On items where the geometric form was already familiar—a circle on item 3, a 45°-45°-90° triangle on item 4—mental visualization proved largely sufficient, and both subjects produced essentially correct symbolic work, although AR-7's item-3 notation contained a literal writing slip (140/20 instead of 140/2) that, taken at face value, would be mathematically inconsistent, later clarified as unintentional. By contrast, on items requiring the student to construct a visual representation from scratch (item 1) or to check all four alternatives exhaustively (item 5), both subjects relied on unverified visual impression, leading to incomplete or partially incorrect conclusions; on item 6, both subjects' identification of the correct dice nets was incomplete or wrong when judged from the written response alone, though follow-up interview with AR-8 revealed that the correct nets (II and III) had in fact been identified mentally but never written down. Overall, the AR profile is characterized by a marked gap between the conceptual understanding elicited through interview and the representation actually documented on paper: representation was treated as optional rather than as an integral communicative and verificatory tool, so that errors of omission—rather than of understanding—were the dominant source of imperfect scores.

Cross-Case Summary and Triangulation

Table 6 synthesizes the representational profile of each thinking-style group across the dominant sequence of representation use, the functional role of each representation type, and the group's principal limitation.

Table 6. Summary of Representation Profile Characteristics by Gregorc Thinking Style

Style	Dominant sequence	Role of visual representation	Role of symbolic representation	Main limitation
CS	Verbal → Visual → Symbolic → Verbal, applied linearly on nearly every item	Active source of calculation-relevant information	Documented in a stepwise, systematic manner	Occasional gap between mental understanding and written documentation
AS	Verbal (concept) → Visual (confirmation) → Symbolic (formalization)	Confirms an already-formed mental model	Favors exact, general-formula-first procedures	Elimination decisions generalized without exhaustive numerical verification
CR	Visual (exploration) → Symbolic/Verbal follow inconsistently	Dominant; often extends to physical manipulatives	Successful mainly when the visual model already contains complete information	Difficulty transforming a correct visual model into an accurate symbolic procedure
AR	Mental visualization/intuition → selective written representation	Used selectively; often internal rather than externalized	Adequate on familiar geometric forms; prone to incomplete final steps	Considerable gap between interview-elicited understanding and documented, verifiable work

Cross-checking the mathematics-representation test results against the semi-structured interviews for all four thinking-style groups showed consistent convergence: patterns evident in students' written work—verbal organization preceding calculation for CS, concept-first reasoning for AS, visual-led exploration for CR, and reliance on mental visualization for AR—were corroborated by participants' own accounts of their problem-solving process, and were further supported by information obtained from the participating mathematics teacher. This convergence across independent data sources supports the credibility of the representational profiles reported above.

Discussion

Concrete Sequential Thinkers

Concrete Sequential (CS) students demonstrate a systematic, highly integrated deployment of verbal, visual, and symbolic representations structured in a distinct linear sequence: initiating with verbal organization of given parameters, constructing a visual model of the geometric context, executing symbolic calculations, and concluding with a verbal synthesis. This ordered progression indicates that verbal representation serves a crucial metacognitive function—clarifying and structuring thought prior to operational processing—which corroborates the framework of Roorda et al. (2024) regarding the explanatory and clarifying roles of verbalization in problem-solving. Furthermore, visual representations function as active, informative tools for spatial processing within the space and shape domain, consistent with Žakelj and Klančar (2022) on visual representation clarifying object relationships. Observed variations in the thoroughness of written symbolic work between CS subjects stem from differences in documentation habits rather than underlying conceptual deficits, highlighting the methodological necessity of triangulating written test artifacts with clinical interview data to accurately evaluate representational competence.

These findings extend and refine prior research on cognitive styles in mathematics education by detailing the temporal dynamics of representational integration (Wahab et al., 2026). While Sugiarti et al. (2022) documented that systematic thinkers integrate verbal, visual, and symbolic modalities in spatial problem-solving, the present results demonstrate that CS learners utilize these forms in a mutually reinforcing, highly predictable sequence. Additionally, the results corroborate the findings of Novitasari et al. (2021) regarding Sequential thinkers' proficiency across all three representational dimensions, while revealing how these modalities are operationalized chronologically during problem-solving. Finally, these observations align with the broader characterization of CS thinkers as step-oriented, linear information processors supporting the conclusion of Fauzi et al. (2020) that the structured working habits characteristic of Concrete Sequential students strongly underpin robust mathematical performance.

Abstract Sequential Thinkers

Abstract Sequential (AS) students exhibit a distinct representational profile characterized by prioritizing conceptual understanding prior to procedural execution, distinguishing them from the step-ordered orientation typical of Concrete Sequential (CS) learners. In problem-solving contexts, AS students routinely articulate the underlying rationale behind selected formulas and formulate general rules prior to numerical substitution, utilizing visual representations primarily to validate pre-established conceptual models rather than to extract new information. This behavioral pattern aligns with the findings of Anwar and Rahmawati (2017) and Rahmawati et al. (2017) regarding the explanatory function of verbal representations, as well as established descriptions of AS thinkers as analytical, logical, and conceptually oriented processors. Their commitment to conceptual precision is further illustrated by a marked preference for exact symbolic expressions over decimal approximations. Nevertheless, observed lapses—such as generalizing elimination decisions based on visual impressions rather than exhaustive numerical checks—demonstrate that a robust conceptual orientation does not inherently guarantee rigorous empirical verification.

These findings extend current educational literature by providing a nuanced account of how cognitive style modulates representational integration. While the observed seamless integration of diverse representational modalities aligns with the results of Sugiarti et al. (2022) regarding high-ability learners, the present evidence demonstrates that for AS students, this integration is fundamentally governed by thinking style rather than cognitive ability alone. Furthermore, the results reinforce the observations of Nurhami et al. (2022), who highlighted the coherent, logical, and complete nature of mathematical communication among AS learners. Conversely, the strong performance of AS students across representation-integration measures contrasts with the findings of Fauzi et al. (2020), who reported lower fulfillment of critical-thinking indicators among AS students relative to CS peers. This divergence suggests that performance variations across studies stem from structural differences in the evaluated constructs—namely, mathematical representation ability versus critical thinking skills—rather than an underlying empirical contradiction.

Concrete Random Thinkers

CR students' representation profiles were dominated by visual representation as an exploratory entry point into each problem, consistent with the characterization of concrete random thinkers as learners who favor direct experience, trial of multiple alternatives, and exploratory approaches to problem-solving. CR-6's use of a physical paper cube to verify dice-net items exemplifies this exploratory, hands-on tendency, echoing Maulani et al.'s (2022) on concrete random students' preference for trial-and-error over prescribed procedure.

The central limitation observed for this group was an inconsistent capacity to transform a correct visual model into an accurate symbolic procedure, evident in both subjects' incomplete Pythagorean derivations on the roof-area item and their reliance on unverified visual impression on the garden-fencing item. Interview data indicated that these lapses stemmed from insufficient care in extending visual insight into formal procedure rather than from a lack of conceptual understanding—a pattern consistent with Safitri and Khotimah (2023), who found concrete-oriented students able to use visual representation to understand a problem without always translating that understanding into an optimal solution procedure. This finding diverges from Sugiarti et al.'s (2022), whose higher-ability students balanced all three representation types; here, visual representation remained dominant while its integration with symbolic representation remained underdeveloped.

Abstract Random Thinkers

AR students' representation profiles were marked by heavy reliance on mental visualization and intuitive judgment, consistent with descriptions of abstract random thinkers as reflective, intuitive, holistic processors who attend to overall impression before detail. Both subjects used visual representation selectively—drawing only when a situation was judged genuinely difficult to picture mentally—and left a considerable share of their symbolic reasoning undocumented so that omitted steps, rather than conceptual misunderstanding, were the recurring source of imperfect answers, as on the roof-area item where both subjects' Pythagorean procedures stopped one step short of the required answer.

This finding is consistent with Nurhami et al.'s (2022), who reported that abstract random students explain ideas well orally without necessarily documenting their process or conclusions in writing, and with Rahmatika et al.'s (2022), who found that students inclined toward rapid decision-making often forgo thorough verification—a tendency mirrored here in AR students' incomplete checking on the garden fencing and dice net items. By contrast with Sugiarti et al. (2022), whose higher-ability students achieved balanced integration of all three representation forms, the AR students in this study did not achieve comparable integration, since much of their reasoning process remained confined to mental representation rather than being externalized in a form that could be verified.

Implications, Limitations, and Future Research

The differentiated representational profiles identified in this study emphasize the necessity of targeted instructional practices: Sequential-oriented learners thrive when structured, stepwise methodologies are reinforced, Abstract Random students benefit from scaffolding that prompts the externalization of internal mental processes, and Concrete Random individuals require tasks that explicitly bridge visual insights with formal symbolic execution. These pedagogical insights, however, should be evaluated within the context of the study's scope, as its qualitative case-study designing eight participants from a single institution—precludes statistical generalization, and the empirical evaluation was limited to the PISA Space and Shape domain. Consequently, future investigations should expand to larger, more diverse cohorts, integrate alternative PISA mathematical content domains (such as Quantity, Change and Relationships, and Uncertainty and Data), and adopt longitudinal frameworks to assess the long-term stability of thinking-style-driven representational profiles.

4. CONCLUSION

This study examined mathematical representation ability in solving PISA Space and Shape problems among tenth-grade students at Islamic High Secondary School DDI Takkalasi, viewed through the four Gregorc thinking styles. Concrete sequential students used verbal, visual, and symbolic representation systematically and in a mutually reinforcing sequence, with verbal representation organizing information at the outset and communicating conclusions at the end; visual representation modeling the geometric situation; and symbolic representation executing calculations in a stepwise manner—a pattern that supported strong performance on most items. Abstract Sequential students were characterized by a tendency to secure conceptual understanding before proceeding to computation, using verbal representation to explain and justify reasoning, visual representation to confirm understanding already formed mentally, and symbolic representation to formalize concepts once judged secure; integration across the three representation forms was strong overall, though verification of alternative solutions was not always exhaustive. Concrete Random students relied predominantly on visual representation as an exploratory medium for understanding a problem, building concrete models—including, in one case, a physical paper manipulative—before determining a solution procedure; however, the transformation of visual models into accurate symbolic procedures was not consistently achieved. Abstract Random students depended heavily on mental visualization and intuition, using written representation selectively so that a substantial part of their reasoning process went undocumented on the answer sheet; although interviews revealed reasonably strong conceptual understanding, this understanding was not always translated into complete, verifiable visual, symbolic, and verbal representation, resulting in errors traceable to insufficient documentation and verification rather than to conceptual misunderstanding.

Taken together, these findings indicate that Gregorc's thinking style is systematically associated with distinct profiles of mathematical representation ability and that assessing this ability from written work alone risks underestimating the understanding of students—particularly those with random-oriented thinking styles—whose reasoning is only partly externalized on paper.

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