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Development of Learning Materials Based on the MURDER Collaborative Learning Model to Enhance Students' Conceptual Understanding of Mathematics

Sri Rahmayani[1*], Afnaria[2], Zainal Azis[3]

[1,2]Department of Mathematics Education, Faculty Teacher Trainer and Education, Universitas Islam Sumatera Utara
[3]Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Muhammadiyah Sumatera Utara

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ABSTRACT

The low level of students' conceptual understanding in mathematics highlights the need for innovative and structured learning approaches. This study aims to develop mathematics learning materials based on the MURDER collaborative learning model (*Mood, Understand, Recall, Digest, Expand,* and *Review)* to enhance students' conceptual understanding. The research employed a Research and Development (R&D) design involving expert validation, limited trials, and classroom implementation to evaluate the validity, practicality, and effectiveness of the developed module. The model helps students recall and comprehend mathematical concepts more effectively, supporting both teaching and learning processes in the classroom. The product was validated by three expert groups. Content validation produced an average score of 88.2%, media validation 92.5%, and language validation 85.5%, with an overall mean of 88.7%, indicating that the module is *highly feasible*. Practicality testing, based on student response questionnaires, yielded an average score of 82%, classified as *highly practical*. Effectiveness testing in a Grade 10 science class involving 30 students resulted in a mean score of 83.6%, categorized as *highly effective*. Furthermore, the N-Gain value of 0.81demonstrates a *high* improvement in students' conceptual understanding after using the module. These results confirm that the MURDER-based mathematics learning module is valid, practical, and effective for improving students' conceptual understanding, particularly in learning the sine and cosine rules. The developed module can serve as a reference for schools and educators seeking to implement collaborative and conceptually oriented mathematics learning.

Corresponding Author:

Sri Rahmayani,
Department of Mathematics Education,
Faculty Teacher Trainer and Education,
Universitas Islam Sumatera Utara
Jl. Karya Bakti No. 34, Kelurahan P. Masyhur, Kecamatan Medan Johor, Sumatera Utara, Indonesia.
Email: sryrahmayani11@gmail.com

Introduction

Mathematics is a fundamental discipline that plays an essential role in the advancement of science and technology (Melhuish et al., 2020; Thanheiser & Melhuish, 2023). It serves as the foundation for reasoning, logical thinking, and problem-solving across all areas of knowledge (Johnson et al., 2017; Mata-Pereira & da Ponte, 2017). In Indonesia, mathematical competence is one of the main objectives of the national education curriculum. The National Education System Law No. 20 of 2003 defines education as a conscious and planned effort to develop students' potential holistically, spiritually, intellectually, and socially. Mathematics, therefore, contributes not only to intellectual development but also to the formation of disciplined, analytical, and reflective individuals (Martinez-Lorca et al., 2023). This establishes the philosophical and educational urgency of mathematics as a key component of national education.

Mathematics is taught at every level of schooling (from elementary to senior high school) and receives greater instructional time than other subjects (Kontorovich, 2021; Rich et al., 2019). It remains closely related to everyday human activities and helps students make sense of real-world phenomena through abstract reasoning (Wasserman, 2017). Mathematics is the science of structures, order, and relationships involving fundamental processes such as calculation, measurement, and the representation of objects (Lee et al., 2018). This definition reflects the dual nature of mathematics as both an abstract system and a tool for understanding the world. Thus, mathematics education must equip students not only with procedural skills but also with meaningful conceptual understanding that can be applied in various contexts.

Education, in a broader sense, aims to develop human abilities and personalities through teaching, guidance, and training that promote active interaction with the environment (Adu-Gyamfi et al., 2019). Its ultimate goal is to prepare individuals to face future social and professional challenges. Achieving this goal requires continuous improvement in educational quality through curriculum refinement, enhancement of teacher competencies, and the development of effective learning strategies (Pepin, 2021; Rezat et al., 2021). In this context, improving the quality of instructional materials and learning models becomes an important step toward ensuring meaningful and equitable learning for all students. This creates a direct link between educational reform and innovation in classroom instruction.

In line with efforts to improve learning quality, learning materials play a crucial role in supporting effective instruction. Learning materials are systematically designed resources that help students achieve specific competencies stated in the curriculum. Learning materials as structured content that directs learners toward competency mastery, while Sormunen et al. (2020) emphasizes their central role in teaching and learning, which may take the form of text, images, audio, or video. Dreyfus (2018) highlights the importance of systematic organization, and Hughes et al. (2020) adds that they must be sequential and specific to promote independent learning. Thahir et al. (2019) further explains that learning materials serve as instructional tools that benefit both teachers and students when designed coherently.

To strengthen learning materials, one promising instructional model is the MURDER model, *Mood, Understand, Recall, Digest, Expand,* and *Review.* This model helps students recall and internalize concepts more effectively, fostering both comprehension and retention. Research has shown that MURDER encourages creative thinking through open-ended problem-solving and enhances metacognitive regulation of learning . It is typically implemented in small cooperative groups, promoting active verbal and written communication among students. However, Jossberger et al. (2025) notes that its application still lacks contextual problem integration, which limits students' ability to connect concepts to real-world situations. This highlights the need for adaptation of the model within meaningful and contextualized learning settings.

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v5i4.2422 Volume 5, No 4, October - December 2025, pp. 1520– 1532

In relation to this model, conceptual understanding serves as a key indicator of learning success. It reflects the ability of students to explain mathematical ideas in their own words while maintaining the original meaning (Chorney et al., 2024; Sengupta-irving & Agarwal, 2017). Relational understanding enables learners to apply mathematical principles flexibly and meaningfully to a variety of problems (Lee et al., 2018; Wilkie & Hopkins, 2024). When instruction fails to provide appropriate guidance, misconceptions may persist and low self-efficacy may reduce students' performance and motivation (Khozaei et al., 2022). Thus, strengthening students' conceptual understanding becomes a critical aspect of improving mathematics learning outcomes.

Preliminary observations and interviews with a mathematics teacher (Mrs. Dewi) at SMAN 1 Batang Natal revealed that many students struggled to grasp mathematical concepts. The main instructional material used was a standard textbook, which presented content too briefly and lacked elaboration to foster deeper comprehension. During classroom observations, students appeared disengaged; some talked to peers or used mobile phones during lessons. This situation indicated low learning motivation and insufficient understanding of mathematical concepts. Therefore, the existing materials and strategies are not yet effective in supporting conceptual learning in mathematics.

Given these challenges, there is an urgent need to implement an instructional model that can enhance students' motivation and conceptual understanding. The MURDER collaborative learning model, which integrates cognitive, metacognitive, and affective elements, offers a comprehensive framework for improving comprehension and engagement. By emphasizing structured learning phases, it provides students with opportunities to manage their emotions, monitor their understanding, and review knowledge meaningfully. Consequently, this study aims to develop and evaluate learning materials based on the MURDER model to improve students' conceptual understanding of mathematics, particularly in the topic of the sine and cosine rules.

Method

Type of Research

This study employed a Research and Development (R&D) design aimed at producing a valid, practical, and effective learning product and testing its effectiveness in improving students' conceptual understanding. The developed product was a mathematics learning module based on the MURDER collaborative learning model, Mood, Understand, Recall, Digest, Expand, and Review, focused on the topic of the sine and cosine rules. The research adopted the 4-D modelproposed, which includes four systematic stages: Define, Design, Develop, and Disseminate. This formative type of research was intended not only to produce an instructional product but also to refine it through iterative validation and testing to ensure quality and usability.

Research Subjects

The study was conducted at SMAN 1 Batang Natal, located in Mandailing Natal Regency, North Sumatra, Indonesia. The population comprised 120 students of Grade 10 during the 2023/2024 academic year. Two classes were selected as research samples: Class Q (30 students) and Class P (28 students). Class Q served as the experimental group, which received instruction using the MURDER-based learning module, while Class P served as the

control group, taught using the conventional method. Data were also obtained from a mathematics teacher and two subject-matter experts who participated as validators.

Instruments

Several research instruments were employed to gather both quantitative and qualitative data. Validation sheets were used to evaluate the validity of the developed MURDER-based mathematics learning module, with assessments conducted by experts in content, media, and language. Student and teacher response questionnaires were administered to determine the practicality of the module during classroom implementation. To measure the improvement in students' conceptual understanding, pre-tests and post-tests were given before and after the learning process. In addition, observation sheets and interview guides were utilized during the Define stage to identify instructional needs, learning difficulties, and classroom conditions. All rating instruments adopted a five-point Likert scale, ranging from 1 (Strongly Disagree) to 5 (Strongly Agree), following the measurement standards proposed.

Procedures

The research procedures were carried out through four systematic stages based on the 4-D development model: Define, Design, Develop, and Disseminate. In the Define stage, the researcher identified learning problems through classroom observations and teacher interviews, analyzed student characteristics, clarified learning objectives, and determined the content scope related to the sine and cosine rules. The Design stage involved preparing test instruments, organizing the sequence of learning materials, designing the module layout, and developing a prototype aligned with the learning objectives and indicators. During the Develop stage, expert validations of both the instruments and the learning module were conducted, followed by revisions based on expert feedback. Field trials were then implemented to examine the module's practicality and effectiveness by administering pre-tests and post-tests, collecting student responses, and obtaining teacher and expert evaluations. Finally, in the Disseminate stage, the validated module was packaged into a printed guidebook and introduced to teachers and schools through limited distribution, accompanied by documentation and user feedback evaluation to guide further refinement and adoption.

Analysis

The data analysis in this study consisted of four main components: validity, practicality, effectiveness, and conceptual understanding. The validity analysis was based on expert evaluations using the formula

$$P = \frac{n}{N} \times 100\%$$

where P represents the percentage of validity, n is the total score obtained, and N is the ideal maximum score. The interpretation followed Sugiyono's (2014) classification, in which 81–100% indicates Highly Valid, 61–80% Valid, 41–60% Moderate, 21–40% Less Valid, and 0–20% Invalid.

The practicality analysis was determined from student and teacher response questionnaires using the formula

$$P = \frac{\text{Total obtained score}}{\text{Maximum possible score}} \times 100\%$$

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
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Scores ranging from 81–100% were classified as Highly Practical, 61–80% as Practical, 41–60% as Moderately Practical, 21–40% as Less Practical, and below 20% as Not Practical .

The effectiveness analysis employed Akbar's (2017) formula

$$NPr = \frac{TSe}{TSmax} \times 100\%$$

where TSe is the total score obtained by students and TSmax is the maximum score. The results were categorized as Highly Effective (81–100%), Effective (61–80%), Less Effective (41–60%), and Not Effective (below 40%).

The conceptual understanding analysis used the normalized gain (N-Gain) formula by:

$$N_{gain} = \frac{S_{post} - S_{pre}}{100 - S_{pre}}$$

where S_{pre} and S_{post} represent the average pre-test and post-test scores, respectively. The N-Gain values were interpreted as High (N-g > 0.7), Moderate (0.3 ≤ N-g ≤ 0.7), and Low (N-g < 0.3). All quantitative data were processed using Microsoft Excel, while qualitative data from interviews and observations were analyzed descriptively to strengthen and contextualize the interpretation of the findings.

Results

This research was conducted at SMAN 1 Batang Natal, located in Mandailing Natal Regency, North Sumatra, with Grade 10 science students (n = 30). The study adopted a Research and Development (R&D) approach using the 4-D model (Define, Design, Develop, and Disseminate) by Thiagarajan et al. (1974). The product developed was a mathematics learning module based on the MURDER collaborative learning model (Mood, Understand, Recall, Digest, Expand, Review) focused on the sine and cosine rules. The research aimed to test the validity, practicality, and effectiveness of the module, as well as its impact on students' conceptual understanding. The study took place over six weeks during the 2023/2024 academic year.

Three groups of experts evaluated the validity of the developed MURDER-based mathematics module, consisting of content experts, media experts, and language experts. The content experts provided an average score of 88.2%, indicating that the module's material, structure, and alignment with learning objectives were highly appropriate. The media experts awarded a mean score of 92.5%, showing that the design, layout, and visual presentation of the module were clear, engaging, and effectively supported learning activities. Meanwhile, the language experts gave a score of 85.5%, suggesting that the language used in the module was communicative, grammatically correct, and suitable for the students' level of comprehension. Overall, the module achieved a mean validity score of 88.7%, which falls under the Highly Validcategory based on Sugiyono's (2014) criteria. These results confirm that the developed module meets academic and pedagogical standards and is ready for implementation in classroom learning.

Table 1. Expert Validation Results

Aspect Evaluated	Expert 1	Expert 2	Mean (%)	Category
Content	88.2	88.2	88.2	Highly Valid
Media	92.0	93.0	92.5	Highly Valid
Language	86.0	85.0	85.5	Highly Valid
Overall Mean			**88.7**	**Highly Valid**

These results indicate that the module content aligns well with curriculum standards, the design layout and visuals are clear and attractive, and the language used is communicative and age-appropriate. The addition of multimedia links in the Mood phase and the inclusion of more contextual examples were among the key improvements suggested by validators. After revisions, experts agreed that the MURDER-based module could be implemented in classroom instruction effectively.

The practicality test was conducted by distributing student response questionnaires to 30 students after the trial phase. Each questionnaire contained ten statements rated on a five-point Likert scale. The total mean score was 82%, classified as Highly Practical (see Table 2).

Table 2. Practicality Scores from Student Responses

Range (%)	Category	Mean Score (%)	N (Students)
81–100	Highly Practical	**82.0**	30

Students reported that the learning instructions were easy to follow, the module helped them recall previous material, and the examples were relevant to real-life situations. Teachers also observed that the six-step structure (Mood–Understand–Recall–Digest–Expand–Review) facilitated classroom management and group interaction. These findings demonstrate that the developed module is user-friendly and suitable for both individual and collaborative learning settings.

Effectiveness was determined using achievement tests (post-test) administered after the module implementation. Each test contained four problem-solving items related to the sine and cosine rules. The mean score obtained by students was 83.6%, which falls into the Highly Effective category (see Table 3).

Table 3. Summary of Student Achievement Scores

Mean Score (%)	Standard Category	Description
83.6	81–100	Highly Effective

The result indicates that students were able to apply both the sine and cosine rules accurately in various contextual problems. Compared with the pre-test results (mean = 42.5%), students' understanding of geometric relationships improved significantly. This suggests that the module not only enhanced procedural fluency but also strengthened conceptual understanding. Teachers noted fewer computational errors and more logical explanations in student answers, especially in determining the correct rule based on triangle type. To measure improvement in conceptual understanding, the normalized gain (N-Gain) was calculated using formula. The average pre-test score was 42.47, and the average post-test score was 79.53, resulting in an N-Gain value of 0.81 (see Table 4).

Table 4. N-Gain Results of Students' Conceptual Understanding

Variable	Pre-test Mean	Post-test Mean	N-Gain	Category
Conceptual Understanding	42.47	79.53	**0.81**	High

The N-Gain value of 0.81 indicates a high level of improvement in conceptual understanding after using the MURDER-based module. Students could now explain the relationship between sides and angles in non-right triangles, determine when to use the sine or cosine rule, and justify their solutions. The structured phases, particularly Digest and Expand, helped students integrate new concepts into prior knowledge. This suggests that metacognitive reflection within the module supported deeper conceptual reasoning.

When analyzed collectively, the validity, practicality, effectiveness, and N-Gain data confirm that the developed module meets the criteria for a high-quality learning product. The validity score (88.7%) demonstrates content alignment and expert approval, while the

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v5i4.2422 Volume 5, No 4, October - December 2025, pp. 1520– 1532

practicality score (82%) shows ease of use and learner engagement. The effectiveness score (83.6%) confirms positive learning outcomes, and the N-Gain of 0.81 evidences significant cognitive improvement. Together, these findings imply that integrating the MURDER learning model into trigonometry lessons can meaningfully enhance students' conceptual understanding through structured reflection and collaboration.

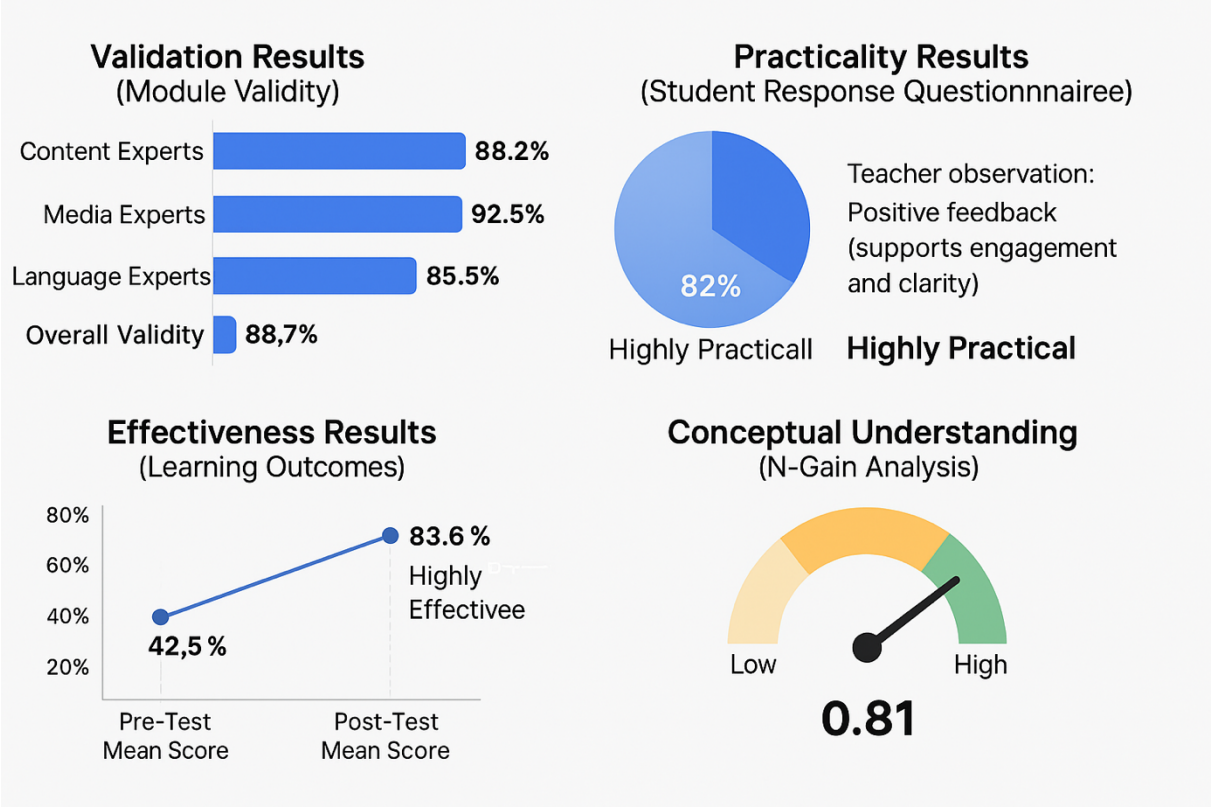

Figure 1. Research Findings

Table 5. Summary of Research Findings

Aspect Evaluated	Indicator of Measurement	Mean Score (%)	Category	Interpretation
Validity	Expert judgment (Content, Media, Language)	88.7	Highly Valid	Module meets quality and content standards
Practicality	Student response questionnaire	82.0	Highly Practical	Easy to use and engaging
Effectiveness	Post-test achievement	83.6	Highly Effective	Improves student performance
Conceptual Understanding	N-Gain Analysis	0.81	High	Significant improvement in conceptual mastery

The results summarized in the Table 5 anda Figure 1 demonstrate that the MURDER-based mathematics learning module fulfills all major criteria for a high-quality educational product. The high validity score confirms that the module's content, media, and language align well with pedagogical standards and learning objectives. The practicality score shows that both students and teachers found the module easy to implement and engaging during classroom use. The effectiveness data, supported by strong post-test results, indicate that the module successfully enhanced students' performance in applying the sine and cosine rules. Finally, the

high N-Gain value highlights a substantial improvement in students' conceptual understanding, suggesting that the structured MURDER phases effectively facilitated deeper and more meaningful learning. Collectively, these findings affirm that the developed module is suitable for broader implementation in mathematics education to foster both conceptual mastery and active student participation.

Discussion

The results of this study reveal that the MURDER-based mathematics learning module does more than improve students' scores, it transforms the way they engage with mathematical ideas. The validation process confirmed that the module's design successfully integrates clarity, coherence, and visual appeal. Experts noted that the structure of explanations and examples supports gradual concept formation, enabling students to move beyond memorization toward understanding. The language and layout encourage active exploration rather than passive reading, which is crucial for fostering curiosity and independent reasoning in mathematics learning. In classroom implementation, both teachers and students experienced a shift in learning dynamics. The module helped create a more interactive and reflective classroom atmosphere, where students were not merely recipients of information but participants in constructing meaning. Teachers reported that the MURDER sequence guided the flow of lessons naturally, from emotional readiness to deeper conceptual reflection. The Mood and Understand phases, for instance, played an essential role in focusing students' attention and preparing them to engage cognitively (Thahir et al., 2019). The Recall and Digestphases supported reconstruction of prior knowledge, while Expand and Review nurtured self-assessment and collaboration. This cyclical movement aligns with theories of metacognition, where learning occurs through continuous monitoring, regulation, and evaluation of thought processes.

One of the most significant transformations observed lies in students' reasoning. Before the module's use, many tended to apply formulas mechanically. During the learning process, however, students began articulating the logic behind their choices, why a sine rule fits certain triangle conditions or how cosine connects to spatial relations (Chorney et al., 2024; Kaiser et al., 2025; Sengupta-irving & Agarwal, 2017). This change suggests that the module not only improved comprehension but also cultivated relational understanding, a hallmark of genuine mathematical thinking. Such reasoning development reflects the essence of conceptual learning: students internalize structures of thought, not just procedural patterns.

The collaborative design of the module also played a crucial role. Working in small groups encouraged peer explanation, negotiation of meaning, and the correction of misconceptions (Sormunen et al., 2020). Through this interaction, students' understanding became more stable and transferable (Walkington et al., 2024; Xie & Masingila, 2017). The teacher's role shifted toward facilitation, asking probing questions, guiding reflection, and providing scaffolds when necessary (Losano et al., 2018; Olsher et al., 2025). This approach resonates with social constructivist principles, where knowledge emerges through dialogue and shared problem-solving. The MURDER model, therefore, does not simply deliver content; it orchestrates a learning culture that values exploration, communication, and mutual understanding.

Another noteworthy insight concerns students' attitudes. Many learners who previously viewed trigonometry as abstract and intimidating began to approach it with curiosity. The Mood phase, which involves emotional readiness and motivation, seemed to reduce anxiety and create a sense of ownership over the learning process (Khozaei et al., 2022; Rogowska et al., 2022). When emotional engagement is sustained throughout the lesson, cognitive engagement naturally deepens. Students started to associate mathematical activity

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
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with satisfaction and confidence rather than pressure and confusion. This emotional-cognitive balance, often overlooked in traditional instruction, became a key driver of conceptual progress.

Finally, the overall learning process demonstrated that structured reflection is indispensable in mathematics education. The final Review stage allowed students to re-examine their reasoning, identify conceptual gaps, and articulate new insights. This reflective closure helped solidify their understanding and prepare them for independent application. The emphasis on reflection distinguishes the MURDER model from conventional task-oriented methods (Kaiser et al., 2025; Liljekvist et al., 2017). It bridges the gap between doing mathematics and thinking mathematically, positioning reflection as both the means and the goal of learning. In essence, the implementation of the MURDER model redefined learning as a dialogic and reflective process. The module empowered students to think critically, express reasoning clearly, and view mathematics as a meaningful human activity (Simon et al., 2018). It demonstrated that effective learning design does not depend solely on the quantity of exercises or content coverage but on the quality of cognitive engagement it promotes. This depth of engagement (emotional, social, and intellectual) represents the true success of the module and signals a promising direction for future mathematics pedagogy.

Conclusion

This study demonstrates that the development and implementation of a MURDER-based mathematics learning module can significantly enhance the depth and quality of students' learning experiences. The module, structured around the six interrelated phases proved to be more than a sequence of instructional steps; it became a framework for cultivating reflective, collaborative, and conceptually rich learning. Through its use, students were guided to think more critically, articulate their reasoning, and connect abstract trigonometric concepts with meaningful understanding. The study found that the MURDER model effectively integrates cognitive and metacognitive processes. Students were not only able to apply mathematical procedures but also to explain the logic behind them. This shift from procedural repetition to relational comprehension represents a transformation in the way learners internalize mathematical ideas. The emotional readiness fostered during the *Mood* phase and the self-reflection embedded in the *Review* phase supported a continuous process of awareness, adjustment, and understanding.

Furthermore, the collaborative design of the module encouraged peer dialogue and shared construction of meaning, enabling students to correct misconceptions and strengthen reasoning through interaction. Teachers, in turn, were able to transition from the role of information providers to facilitators of thought, guiding students' inquiry and reflection. This collaborative metacognitive structure nurtured both intellectual growth and a positive disposition toward mathematics learning. Overall, the MURDER-based module has proven to be a valid, practical, and effective innovation for mathematics instruction, particularly for topics requiring conceptual precision such as the sine and cosine rules. Its emphasis on emotional engagement, structured reflection, and collaborative reasoning offers a model for fostering higher-order thinking in mathematics classrooms. The study concludes that sustainable improvement in mathematical understanding emerges not from increased content exposure, but from learning environments that make students active participants in constructing, testing, and refining their own ideas.

Conflict of Interest

The authors declare that there is no conflict of interest.

Authors' Contributions

S.R., A., and Z.A. jointly contributed to the conception and design of the study, data collection, analysis, and manuscript preparation. The three authors worked collaboratively and proportionally in all stages of the research process, including drafting, revising, and finalizing the article. All authors reviewed and approved the final version of the manuscript before submission. The percentage of contribution in conceptualization, drafting, and revision is as follows: S.R.: 60 %, A.: 20%, and Z.A.: 20 %.

Data Availability Statement

The authors declare that the data supporting the findings of this study will be made available by the corresponding author, [S.R.], upon reasonable request.

References

Adu-Gyamfi, K., Schwartz, C. S., Sinicrope, R., & Bossé, M. J. (2019). Making sense of fraction division: domain and representation knowledge of preservice elementary teachers on a fraction division task. *Mathematics Education Research Journal*, *31*(4), 507–528. https://doi.org/10.1007/s13394-019-00265-2

Chorney, S., Evans, K. R., & Staples, M. (2024). Conceptualizing reasoning practices in the context of sociomathematical issues. *Journal of Mathematical Behavior*, *73*. https://doi.org/10.1016/j.jmathb.2024.101124

Dreyfus, T. (2018). Learning Through Activity – Basic research on mathematical cognition. *Journal of Mathematical Behavior*, *52*(November 2017), 216–223. https://doi.org/10.1016/j.jmathb.2018.04.001

Hughes, E. M., Riccomini, P. J., & Lee, J. (2020). Investigating written expressions of mathematical reasoning for students with learning disabilities. *Journal of Mathematical Behavior*, *58*(February), 100775. https://doi.org/10.1016/j.jmathb.2020.100775

Johnson, H. L., McClintock, E., & Hornbein, P. (2017). Ferris wheels and filling bottles: a case of a student's transfer of covariational reasoning across tasks with different backgrounds and features. *ZDM - Mathematics Education*, *49*(6), 851–864. https://doi.org/10.1007/s11858-017-0866-4

Jossberger, H., Boshuizen, H. P. A., Gruber, H., & Krauss, S. (2025). Trends in expertise studies in the domain of teaching. *ZDM - Mathematics Education*. https://doi.org/10.1007/s11858-025-01745-5

Kaiser, G., König, J., Krauss, S., Buchholtz, N., & Gruber, H. (2025). Are teacher expertise and teacher competence complementary or contradictory research paradigms? Reflections on a possible integration and first conceptualizations. *ZDM - Mathematics Education*. https://doi.org/10.1007/s11858-025-01743-7

Khozaei, S. A., Zare, N. V., Moneghi, H. K., Sadeghi, T., & Taraghdar, M. M. (2022). Effects of quantum-learning and conventional teaching methods on learning achievement, motivation to learn, and retention among nursing students during critical care nursing education. *Smart Learning Environments*, *9*(1). https://doi.org/10.1186/s40561-022-00198-7

Kontorovich, I. (2021). Minding mathematicians' discourses in investigations of their feedback on students' proofs: a case study. *Educational Studies in Mathematics*, *107*(2), 213–234. https://doi.org/10.1007/s10649-021-10035-2

Kognitif: Jurnal Riset HOTS Pendidikan Matematika
https://doi.org/10.51574/kognitif.v5i4.2422 Volume 5, No 4, October - December 2025, pp. 1520– 1532

Lee, K., Ng, S. F., & Bull, R. (2018). Learning and solving algebra word problems: The roles of relational skills, arithmetic, and executive functioning. *Developmental Psychology*, *54*(9). https://doi.org/10.1037/dev0000561

Liljekvist, Y., Mellroth, E., Olsson, J., & Boesen, J. (2017). Conceptualizing a Local Instruction Theory in Design Research: Report from a Symposium. *Swedish Society for Research in Mathematics Education*, 119–128. http://ncm.gu.se/media/smdf/Published/No11_Madif10/119128_madif_SY003_liljekvist.pdf

Losano, L., Fiorentini, D., & Villarreal, M. (2018). The development of a mathematics teacher's professional identity during her first year teaching. *Journal of Mathematics Teacher Education*, *21*(3), 287–315. https://doi.org/10.1007/s10857-017-9364-4

Martinez-Lorca, M., Criado-Álvarez, J. J., Romo, R. A., & Martinez-Lorca, A. (2023). The impact of mental health, affectivity, emotional intelligence, empathy and coping skills in Occupational Therapy students. *Retos*, *50*. https://doi.org/10.47197/retos.v50.99384

Mata-Pereira, J., & da Ponte, J. P. (2017). Enhancing students' mathematical reasoning in the classroom: teacher actions facilitating generalization and justification. *Educational Studies in Mathematics*, *96*(2), 169–186. https://doi.org/10.1007/s10649-017-9773-4

Melhuish, K., Thanheiser, E., & Guyot, L. (2020). Elementary school teachers' noticing of essential mathematical reasoning forms: justification and generalization. In *Journal of Mathematics Teacher Education* (Vol. 23, Issue 1). Springer Netherlands. https://doi.org/10.1007/s10857-018-9408-4

Olsher, S., Abdu, R., & Shalata, M. (2025). The relationships between student content-specific grouping and teachers-students' interactions during online collaborative mathematical task solving. *Educational Studies in Mathematics*, *119*(2), 249–268. https://doi.org/10.1007/s10649-024-10382-w

Pepin, B. (2021). Connectivity in support of student co - design of innovative mathematics curriculum trajectories. *ZDM – Mathematics Education*, *53*(6), 1221–1232. https://doi.org/10.1007/s11858-021-01297-4

Rezat, S., Fan, L., & Pepin, B. (2021). Mathematics textbooks and curriculum resources as instruments for change. *ZDM – Mathematics Education*, *53*(6), 1189–1206. https://doi.org/10.1007/s11858-021-01309-3

Rich, K. M., Yadav, A., & Schwarz, C. V. (2019). Computational thinking, mathematics, and science: Elementary teachers' perspectives on integration. *Journal of Technology and Teacher Education*, *27*(2), 165–205.

Rogowska, A. M., Tataruch, R., Niedźwiecki, K., & Wojciechowska-Maszkowska, B. (2022). The Mediating Role of Self-Efficacy in the Relationship between Approach Motivational System and Sports Success among Elite Speed Skating Athletes and Physical Education Students. *International Journal of Environmental Research and Public Health*, *19*(5). https://doi.org/10.3390/ijerph19052899

Sengupta-irving, T., & Agarwal, P. (2017). Conceptualizing Perseverance in Problem Solving as Collective Enterprise Conceptualizing Perseverance in Problem Solving as Collective. *Mathematical Thinking and Learning*, *19*(2), 115–138. https://doi.org/10.1080/10986065.2017.1295417

Simon, M. A., Placa, N., Kara, M., & Avitzur, A. (2018). Empirically-based hypothetical learning trajectories for fraction concepts: Products of the Learning Through Activity research program. *Journal of Mathematical Behavior*, *52*(October 2017), 188–200. https://doi.org/10.1016/j.jmathb.2018.03.003

Sormunen, K., Juuti, K., & Lavonen, J. (2020). Maker-Centered Project-Based Learning in Inclusive Classes: Supporting Students' Active Participation with Teacher-Directed Reflective Discussions. *International Journal of Science and Mathematics Education*, *18*(4), 691–712. https://doi.org/10.1007/s10763-019-09998-9

Thahir, A., Komarudin, Hasanah, U. N., & Rahmahwaty. (2019). MURDER learning models and self efficacy: Impact on mathematical reflective thinking ability. *Journal for the Education of Gifted Young Scientists*, *7*(4), 1120–1133. https://doi.org/10.17478/jegys.594709

Thanheiser, E., & Melhuish, K. (2023). Teaching routines and student-centered mathematics instruction: The essential role of conferring to understand student thinking and reasoning. *Journal of Mathematical Behavior*, *70*(December 2022), 101032. https://doi.org/10.1016/j.jmathb.2023.101032

Walkington, C., Nathan, M. J., Hunnicutt, J., Washington, J., & Zhou, M. (2024). New kinds of embodied interactions that arise in augmented reality dynamic geometry software. *Journal of Mathematical Behavior*, *75*(June), 101175. https://doi.org/10.1016/j.jmathb.2024.101175

Wasserman, N. H. (2017). Making Sense of Abstract Algebra: Exploring Secondary Teachers' Understandings of Inverse Functions in Relation to Its Group Structure. *Mathematical Thinking and Learning*, *19*(3), 181–201. https://doi.org/10.1080/10986065.2017.1328635

Wilkie, K. J., & Hopkins, S. (2024). Primary students' relational thinking and computation strategies with concrete-to-symbolic representations of subtraction as difference. *Journal of Mathematical Behavior*, *73*(April 2023), 101121. https://doi.org/10.1016/j.jmathb.2023.101121

Xie, J., & Masingila, J. O. (2017). Examining Interactions between Problem Posing and Problem Solving with Prospective Primary Teachers: A Case of Using Fractions. *Educational Studies in Mathematics*, *96*(1), 101–118. https://doi.org/10.1007/s10649-017-9760-9

Author Biographies

Sri Rahmayani is an undergraduate student in the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Islam Sumatera Utara. Her academic interests focus on the development of innovative learning materials and strategies in mathematics education, particularly those aimed at improving students' conceptual understanding and problem-solving abilities. She is actively involved in research projects related to curriculum innovation and instructional design in mathematics learning.
Email: sryrahmayani11@gmail.com

Afnaria is a lecturer in the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Islam Sumatera Utara. Her areas of expertise include mathematics pedagogy, educational research methodology, and the integration of technology in mathematics instruction. She has guided numerous student research projects and publications in mathematics education and is committed to advancing creative, reflective, and student-centered learning practices.
Email: afnaria@uisu.ac.id

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Zainal Azis is a faculty member in the Department of Mathematics Education, Faculty of Teacher Training and Education, Universitas Muhammadiyah Sumatera Utara. His professional work centers on mathematics curriculum development, educational innovation, and the application of collaborative learning models in secondary education. He has contributed to various academic forums and publications promoting effective teaching practices and continuous professional development for mathematics educators.
Email: zainalazis@umsu.ac.id