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Fifth-Grade Students' Conceptual Understanding of Fractions: A Qualitative Study

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ABSTRACT

Understanding fraction concepts is a fundamental foundation for mathematics learning at the elementary school level; however, students often struggle to connect symbols, visual representations, the characteristics of fraction forms, and operations in contextual problems. This study aimed to describe fifth-grade students' understanding of fraction concepts at UPT SD Negeri 041 Padang. A qualitative descriptive design was employed, involving three participants purposively selected to represent high, moderate, and low mathematics ability categories. Data were collected through a fraction concept understanding test consisting of five essay items and task-based interviews. The test instrument and interview guide were validated by two experts using Aiken's V index, yielding average values of 0.875 and 0.861, respectively. Data were analyzed interactively through data reduction, data display, and conclusion drawing and verification. The credibility of the findings was maintained through method triangulation among written responses, interviews, and field notes. The results revealed three patterns of understanding. Students with stronger understanding were able to explain equivalent fractions, classify mixed fractions, distinguish common fractions from whole numbers, represent fractions visually, and systematically apply fraction operations in contextual problems. Students with partial understanding could verbally explain basic concepts but were inconsistent in classifying fraction forms and translating symbols into visual representations. Students with lower understanding tended to rely on surface features, such as visual similarity or the presence of specific numbers, without fully grasping the quantitative relationship among numerator, denominator, and fraction value. These findings underscore the need for fraction assessment to evaluate students' answers, reasoning, representations, and strategies in an integrated manner. Fraction instruction should connect concrete objects, visual representations, symbols, and contextual problems, while encouraging students to explain and verify their problem-solving strategies.

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Introduction

Understanding of concepts is one of the primary foundations of mathematics learning in primary education. Mathematics learning should not be directed merely at mastering computational procedures; it also needs to help students construct meaning, connect various representations, explain mathematical reasoning, and apply concepts in diverse situations (Aki Murata et al., 2012; Kotto et al., 2022; Nabila & Widjajanti, 2020). This orientation is consistent with the direction of learning in the Merdeka Curriculum, which emphasizes mastery of essential competencies, critical reasoning, problem solving, and student-centered learning (Nugroho et al., 2018). In Phase C mathematics learning, fifth- and sixth-grade students are expected to understand and use number concepts, including fractions, across various representations and problem contexts.

The urgency of strengthening mathematical understanding in Indonesia is also reflected in the results of the 2022 Programme for International Student Assessment (PISA). PISA measures the ability of students aged around 15 years to use mathematical, reading, and scientific knowledge to address real-life problems. In PISA 2022, only 18% of Indonesian students reached at least Level 2 in mathematics, whereas the average among OECD countries reached 69% (Habibi & Suparman, 2020; Tohir, 2019; Zulkardi et al., 2020). Level 2 is regarded as the baseline proficiency level that enables students to begin using mathematics in simple real-life situations. Although this finding does not directly describe the abilities of elementary school students, it indicates the importance of strengthening understanding, reasoning, and the application of mathematical concepts from the early stages of education.

One mathematical concept that is both important and difficult for elementary school students to understand is fractions. Fractions are not only related to the ability to read the symbol $\frac{a}{b}$, but also to understanding that a fraction is a value expressing a quantitative relationship (Eriksson & Sumpter, 2021). In elementary school learning, a fraction can be interpreted as part of a whole, part of a set of objects, the result of division, a measure, a comparison, or an operator (Pedersen & Bjerre, 2021). This diversity of meanings makes fractions a complex concept. Students need to understand the relationship between the numerator and denominator, the concept of a unit or one whole, the equivalence of fraction values, the relationship between fractions and pictorial representations, and the meaning of fraction operations in real contexts.

Students' difficulties in understanding fractions are often shown through a tendency to treat a fraction as two separate whole numbers rather than as a single value (Adu-Gyamfi et al., 2019; Čadež & Kolar, 2018). For example, students may assume that a fraction with a larger denominator always has a larger value, compare fractions based solely on the numerator or denominator, or perform calculations by manipulating the numbers in the numerator and denominator without considering the meaning of the operation. Previous studies have also identified that fraction misconceptions can be grouped into three main categories: misconceptions about basic fraction concepts, misconceptions about comparing and ordering fractions, and misconceptions about fraction operations (Lee & Lee, 2021; Purnomo et al., 2022; Tobias, 2013). These misconceptions can be influenced by epistemological factors, students' learning experiences, the way teachers present concepts, the use of representations, and a tendency toward procedure-focused instruction.

Problems in learning fractions become more complex when students are faced with contextual problems (Qiu & Wu, 2019). In such problems, students are not only asked to calculate but also need to interpret information, determine the appropriate operation, and connect the results of their calculations to the meaning of the given situation. Students who rely solely on memorized procedures often experience difficulty when the form of the problem

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changes, when symbolic representations are replaced with pictures or stories, or when they are asked to justify their answers. Therefore, an incorrect answer to a fraction problem cannot always be understood simply as a computational failure; it may also indicate that the student has not yet built adequate conceptual understanding.

Understanding of fraction concepts needs to be examined in depth because mastery of fractions is an important prerequisite for mathematics learning at the next level. The concept of fractions is closely related to decimals, percentages, ratios, proportions, scale, probability, measurement, and algebra (Copur-Gencturk & Doleck, 2021; MacDonald et al., 2025). Difficulties that go unidentified in elementary school can carry over into later stages of education and hinder the development of students' mathematical abilities. Therefore, teachers need information that not only shows whether students obtained a correct or incorrect answer, but also explains how students interpret fractions, use representations, choose strategies, and construct reasoning when solving problems.

A preliminary study at UPT SD Negeri 041 Padang indicated that some fifth-grade students still experienced difficulty understanding the concept of fractions. Initial information was obtained through communication with the fifth-grade teacher and by administering a contextual essay question about fractions. The question had been discussed with the teacher to ensure its alignment with the fraction material the students had already studied. The question described a situation in which part of a cake had been eaten by a family, after which the remaining cake was divided equally between two neighbors. This situation was designed to explore students' understanding of fractions as part of a whole, the subtraction operation used to determine the remainder, and the division operation used to distribute a portion fairly. In the question, students were informed that $\frac{3}{8}$ of the cake had been eaten. To determine the remaining cake, students should understand that one whole cake can be expressed as $\frac{8}{8}$, so the remaining portion is:

$$\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$$

Next, if $\frac{5}{8}$ of the cake is divided equally between the two neighbors, each neighbor receives:

$$\frac{5}{8} \div 2 = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$$

However, an initial analysis of one student's work revealed a pattern of conceptual error. In the first question, the student wrote $\frac{3}{8}$, the portion that had been eaten, as the answer to the question about the remaining cake. This answer shows that the student had not yet interpreted the concept of "remainder" as the result of subtracting the portion taken from the whole. The student appeared to extract the numerical information given in the problem without connecting it to the mathematical relationship required.

In the second question, the student focused on the phrase "divided by two," but applied it by dividing the denominator or manipulating certain numbers without considering the fraction's value as a single quantity. This response pattern indicates that the student had not yet fully understood fraction division in the context of equal distribution. This error also suggests that the student may still regard the numerator and denominator as numbers that can be treated separately, rather than as components forming a single fraction value. This preliminary finding is consistent with previous studies showing that students often apply rules or procedures mechanically and struggle to distinguish the meaning of a fraction as part of a whole from a fraction as the result of division.

Although research on difficulties in learning fractions has been conducted in various contexts, in-depth research on how students construct an understanding of fractions within a specific school context remains necessary. Each school may have different characteristics, including students' learning experiences, the instructional approaches used by teachers, available media, classroom interaction patterns, and the forms of misconception that develop (Im & Jitendra, 2020; Tan Sisman & Aksu, 2016). Moreover, research that relies solely on achievement tests is not sufficient to explain the reasoning behind students' answers. An analysis of written work, verbal explanations, and students' problem-solving strategies is needed so that patterns of understanding and misconception can be identified more accurately.

This study focuses on exploring fifth-grade students' conceptual understanding of fractions through problems that combine symbolic representation with everyday life contexts. The study examines not only the accuracy of students' answers but also how students interpret information, understand fractions as part of a whole, determine the appropriate operation, use representations, and explain the mathematical reasoning behind their solution strategies. Through a qualitative descriptive approach, this study is expected to provide a more comprehensive picture of the profile of fraction concept understanding and the forms of misconception experienced by fifth-grade students. Based on the above discussion, this study aims to describe fifth-grade students' understanding of fraction concepts at UPT SD Negeri 041 Padang. Specifically, this study seeks to: (1) identify students' ability to interpret fractions as part of a whole; (2) analyze students' ability to use fraction operations to solve contextual problems; (3) identify the forms of errors and misconceptions students make in solving fraction problems; and (4) describe students' thinking strategies when explaining their answers. The results of this study are expected to make a theoretical contribution to the development of research on conceptual understanding and misconceptions of fractions among elementary school students. Practically, the findings can serve as a basis for teachers to design instruction and diagnostic assessments that are more responsive to students' learning needs. For schools, the findings can be used as input to strengthen the quality of mathematics instruction, particularly fraction instruction that emphasizes the use of concrete objects, visual representations, mathematical discussion, and contextual problems. For future researchers, this study can serve as a foundation for developing instructional interventions, misconception diagnostic instruments, or fraction instruction designs based on multiple representations.

Method

Research Design

This study used a qualitative approach with a descriptive design. This design was chosen to gain an in-depth understanding of how fifth-grade students interpret the concept of fractions, use fraction operations, and explain their strategies and reasoning when solving fraction problems. The study was not intended to test the effect of a treatment or produce statistical generalizations, but rather to describe students' thinking processes, error patterns, and indications of misconceptions that emerge in solving fraction problems. The study was conducted within the natural context of classroom learning, without variable manipulation or instructional intervention by the researcher. In this study, the researcher served as the primary instrument, carrying out data collection, interviews, documentation, analysis of written answers, data coding, and conclusion drawing. The use of a qualitative approach allowed the researcher to interpret students' understanding based on evidence from written test results and students' verbal explanations during interviews.

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Context and Participants

The participants in this study were fifth-grade students at UPT SD Negeri 041 Padang. Participants were selected purposively to obtain variation in mathematics ability characteristics. Three students were selected as the main participants, consisting of one student in the high mathematics ability category, one in the moderate category, and one in the low category. The mathematics ability category was determined based on students' mathematics report card scores for the odd semester of the 2025/2026 academic year (Table 1).

Table 1. Mathematics Ability Category Criteria

Mathematics ability score	Ability category
MAS ≤ 60	Low
60 < MAS ≤ 75	Moderate
75 < MAS ≤ 100	High

Note: MAS = students' mathematics ability score.

The participant selection procedure was carried out in several stages. First, the researcher established fifth-grade students at UPT SD Negeri 041 Padang as the research context. Second, the researcher identified students' mathematics ability categories based on their odd-semester mathematics report card scores for the 2025/2026 academic year. Third, one student was selected from each of the high, moderate, and low ability categories. When more than one candidate fell into the same category, selection considered the candidate's ability to communicate ideas, willingness to be interviewed, ease of access during data collection, and the classroom teacher's recommendation. All participants were anonymized in the reporting of results. To protect their identities, participants were coded as ST for the student with high mathematics ability, SS for the student with moderate mathematics ability, and SR for the student with low mathematics ability. This categorization was used to select varied cases, not to determine the level of fraction concept understanding. The determination of conceptual understanding was based on an integrated analysis of test answers and interview results.

Research Instruments

The researcher served as the primary instrument in this study. The researcher directly administered the test, conducted interviews, recorded field notes, transcribed recordings, analyzed data, and interpreted findings. To support the data collection process, the study used several supporting instruments: a fraction concept understanding test (hereafter referred to as the FCUT), an interview guide, and an audio recorder. The FCUT consisted of five essay items. The items were designed to explore students' understanding of fraction material, including the ability to understand fractions as part of a whole, apply fraction operations, connect contextual information to mathematical models, and explain the reasoning behind their answers. Essay-type items were used so that students could demonstrate their thinking process, solution steps, and strategies, rather than only the final answer. The interview guide was used to further explore students' thinking based on their written work. Interviews were conducted in a semi-structured, task-based manner. The researcher asked probing questions based on students' responses, such as the reason for choosing a particular operation, the meaning of the numerator and denominator, the reason for a given answer, and how students connected the context of the problem to the solution procedure. Interviews were conducted after students completed the test, so that the test results could be used as a stimulus to explore the reasoning behind students' answers. In addition, the researcher used a mobile phone as an audio recorder during the interviews. Audio recordings were used to ensure that students' explanations could be transcribed accurately and analyzed in depth. The researcher also took field notes during the test and interviews to document responses, changes in strategy, the use of drawings or additional markings, and other conditions relevant to data interpretation.

Instrument Content Validation

The fraction concept understanding test and the interview guide were validated by two validators or experts. The validation assessed the alignment of the instruments' content with the research objectives and indicators of fraction concept understanding, the clarity of instructions, the readability of the language, the clarity of the items' intent, the appropriateness of the scoring guidelines, and the ability of the interview questions to elicit the required information. The validators' assessments used a five-point scale, ranging from a score of 1 as the lowest rating category to a score of 5 as the highest. Content validity was analyzed using the Aiken's V index with the following formula:

$$\boldsymbol{V} = \frac{\boldsymbol{\Sigma s}}{[\boldsymbol{n}(\boldsymbol{c} - \mathbf{1})]}$$

In this formula, $s = r - l_0$; r is the score given by the validator; l_0 is the lowest score on the rating scale; n is the number of validators; and c is the number of rating categories.

Table 2. Interpretation Criteria for the Aiken's V Index

V value range	Validity category
$0.80 < V \leq 1.00$	Very high
$0.60 < V \leq 0.80$	High
$0.40 < V \leq 0.60$	Moderate
$0.20 < V \leq 0.40$	Low
$0.00 < V \leq 0.20$	Very low

The validation results showed that the fraction concept understanding test had a mean Aiken's V index of 0.875, while the interview guide had a mean Aiken's V index of 0.861. Both instruments therefore had high to very high content validity and were suitable for use in data collection after being revised according to the validators' feedback.

Data Collection Procedure

Data collection was carried out in three stages: preparation, test administration, and task-based interviews. During the preparation stage, the researcher obtained research permits, coordinated with the school principal and the fifth-grade teacher, obtained students' mathematics report card scores, identified prospective participants, validated the instruments, and revised the test and interview guide based on the validators' suggestions. Once the participants were determined, the researcher explained the purpose and procedure of the study to the school, the teacher, the students, and the students' parents or guardians. During the test administration stage, all three participants individually completed five essay items on fraction concept understanding. Students were asked to write down their solution steps and final answers for each item. During the test, the researcher did not provide any hints that could direct students toward a particular answer. The test results were then examined to identify response patterns, strategies, errors, and aspects that needed to be explored further in the interviews. The next stage was an individual, task-based interview. Interviews were conducted after each participant completed the fraction concept understanding test. Students' answer sheets were used as the basis for questions so that students could explain their thinking process in more detail. The

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researcher asked students to explain the reasons for using particular steps, the meaning of the information given in the problem, the reasons for choosing a particular mathematical operation, and the basis for their final answer. All interviews were audio-recorded and transcribed verbatim for analysis. The research data consisted of written test results, interview transcripts, audio recordings, and field notes.

Data Analysis and Credibility of Findings

Data were analyzed using an interactive analysis model comprising three recurring processes: data reduction, data display, and conclusion drawing and verification. During the data reduction stage, the researcher examined each participant's test answers, identified the solution steps used, and flagged answers indicating understanding, procedural errors, or indications of misconception. Interview recordings were transcribed verbatim, after which relevant data were selected and simplified without altering the meaning of students' explanations. Next, the test data and interview transcripts were coded according to the indicators of fraction concept understanding, solution strategies, mathematical reasoning, use of representations, and the types of errors that emerged.

During the data display stage, the analysis results were organized into narrative descriptions, summary tables, interview excerpts, and reproductions or descriptions of students' written work. Data were presented separately for each participant—ST, SS, and SR—so that differences in thinking processes, solution strategies, and error patterns could be clearly compared. Data display also connected evidence from written answers with evidence from the interviews. During the conclusion-drawing stage, the researcher interpreted patterns of fraction concept understanding based on the consistency between test answers and students' verbal explanations. Conclusions were based not only on whether the final answer was correct or incorrect, but also on students' ability to explain the meaning of fractions, determine the appropriate operation, use representations, and provide mathematical reasoning. A student's response was categorized as a misconception when the student demonstrated mathematically inaccurate understanding and that understanding was supported or maintained through verbal explanation during the interview. In contrast, errors resulting only from computational slips, writing mistakes, or carelessness were not directly categorized as misconceptions.

The trustworthiness of the data was maintained through method triangulation. Triangulation was carried out by comparing data from the fraction concept understanding test, the task-based interviews, and the field notes. Test results were used to identify students' response patterns and written strategies, while interviews were used to confirm the reasoning, meaning, and understanding behind those responses. Field notes were used to enrich the understanding of students' responses throughout the research process. A finding was considered credible when there was consistency between the evidence in written answers and students' explanations during the interviews.

Results

This section presents an in-depth interpretation of fraction concept understanding based on five indicators. For each indicator, data from ST, SS, and SR are presented through the connections between written answers, interview excerpts, and the interpretation of students' thinking processes. Images of students' written work are included to show the trace of written reasoning that underlies the interpretation.

Indicator 1: Restating the Concept of Equivalent Fractions

This indicator examines students' ability to interpret equivalent fractions as two or more fractions that differ in form but have the same value. Understanding on this indicator is reflected not only in the ability to write equivalent fractions, but also in the ability to explain the relationship between the numerator, the denominator, and the value of the fraction. ST identified the shaded portion in the first picture as 2/3 and produced 4/6 and 6/9 as equivalent fractions. In the second picture, ST wrote 1/4, 2/8, and 3/12. This response pattern shows that ST understood that a change in the numerator must be followed by a proportional change in the denominator. ST did not simply add numbers at random, but maintained the same ratio across every form of the fraction.

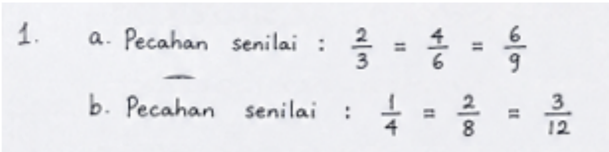

Figure 1. ST's answer on the FCUT, item 1.

> *"Equivalent fractions are fractions that have the same value even though their numerators and denominators are different." (ST, interview, item 1).*
>
> *"I multiply the numerator and denominator by the same number." (ST, interview, item 1).*

Conceptually, the first excerpt shows that ST understood the invariance aspect of equivalent fractions—that a fraction's value remains constant even though the numerator and denominator symbols change. The second excerpt explains the procedural mechanism used to preserve that invariance. The consistency between the definition, the procedure, and the written product shows that ST did not merely memorize the algorithm of multiplying the numerator and denominator, but used it to construct an equivalent fraction value. Nevertheless, ST's explanation remained mainly procedural, since it did not yet touch on the quantitative reason why multiplying the numerator and denominator by the same number preserves the value. In the discussion, this can be interpreted as strong operational conceptual understanding, though it does not necessarily reflect deeper relational justification. To probe the depth of this understanding further, follow-up studies could ask students to explain, using an area model or a number line, why 2/3 equals 4/6.

SS was able to state that equivalent fractions have the same value even though their numerators and denominators differ. The written answer contained a correction or a crossed-out portion, indicating that SS reviewed the answer after comparing it with the picture. This action is significant because it reflects an awareness that the answer needed to be adjusted to match the given representation.

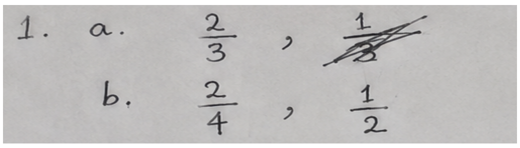

Figure 2. SS's answer on the FCUT, item 1.

> *"Equivalent fractions are fractions that have the same value, even though the numerator and denominator are different." (SS, interview, item 1).*
>
> *"At first I wrote a different answer, but after looking at the picture again, I changed it because I felt it was not quite right." (SS, interview, item 1).*

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Although SS's verbal definition resembles ST's, the quality of their understanding cannot be regarded as equivalent. ST demonstrated consistency between the definition and the strategy used to construct equivalent fractions, whereas SS's answer decision still depended on re-checking against the visual stimulus. Crossing out an answer can reflect a positive monitoring strategy, but it can also indicate that the equivalence relation is not yet fully established within the student's knowledge scheme. SS can therefore be understood as having declarative knowledge of equivalent fractions—able to state the definition—while application and justification of the concept still require representational support. In reporting the results, it should not be concluded that SS held a misconception simply because a correction was made. The data more accurately reflect understanding that is still developing and not yet stable.

SR wrote 1/3 for a picture divided into three equal parts with one part shaded. However, for another picture divided into four equal parts with one part shaded, SR wrote 2/3. The error on the second picture shows that SR had not yet consistently coordinated two key pieces of information: the total number of equal parts as the denominator, and the number of shaded parts as the numerator.

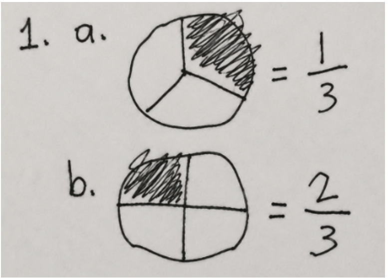

Figure 3. SR's answer on the FCUT, item 1.

> *"Equivalent fractions are fractions whose pictures look almost the same, but I'm still confused about how to find them." (SR, interview, item 1).*
>
> *"Because I saw the circle divided into three parts, and I thought one part was shaded." (SR, interview, item 1).*

The phrase "the pictures look almost the same" shows that SR grounded the meaning of equivalent fractions in perceptual similarity rather than in the relation between values. For SR, two pictures could be considered equivalent if they appeared similar, whereas fraction equivalence is actually determined by the relative quantity represented. This error matters because it can hinder understanding of equivalent fractions when their representations do not look visually similar, such as 1/2 and 4/8. SR's response could be categorized as an indication of misconception if, across several tasks or follow-up questions, the student continued to maintain the belief that visual similarity is the basis of equivalence. If the response occurred only once and the student was able to correct it after using a concrete model, a more cautious interpretation would be an initial conceptual difficulty. This distinction matters so that the article does not label every incorrect answer as a misconception. Cross-subject comparison shows a spectrum of understanding: ST consistently maintained the value and the procedure, SS knew the definition but had not yet shown stable application, while SR had not yet constructed the meaning of equivalence as a value relationship.

Indicator 2: Classifying Mixed Fractions

This indicator evaluates students' ability to use relevant attributes to distinguish mixed fractions from other number forms. Success is determined not only by the number of correct answers, but also by whether students applied consistent criteria across all the objects presented. ST selected 2 1/4 and 5 2/3 as mixed fractions. This choice was accompanied by an explanation that a mixed fraction combines a whole number and a common fraction. ST examined each number form before determining its classification.

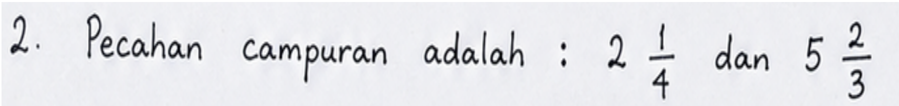

Figure 4. ST's answer on the FCUT, item 2.

"I check whether a number contains both a whole number and a fraction." (ST, interview, item 2).

"I check them one by one. If a number consists of a whole number and a common fraction, then it is a mixed fraction." (ST, interview, item 2).

ST's answer reveals two aspects of understanding. First, the student has a categorical definition of mixed fractions. Second, the student has a systematic checking strategy. The "check them one by one" strategy shows that ST did not rely merely on quick recognition of a familiar form, but applied the same criteria across all the options. This process reduces the likelihood of errors caused by selective attention or by choosing the first option that appears correct. SS selected the forms read as 1 1/3 and 2 1/4 and explained that a mixed fraction has a whole number and a fraction. Verbally, SS used the same attributes as ST. However, the documentation shows notational inconsistency between the list of options and the answer. The accuracy of SS's final answer therefore needs to be verified against the original item image before being judged correct or incorrect.

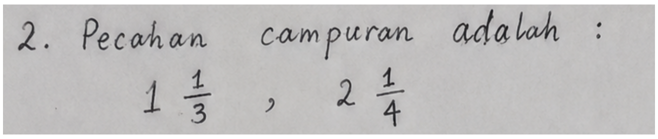

Figure 5. SS's answer on the FCUT, item 2.

"I check whether the number has both a whole number and a fraction." (SS, interview, item 2).

"I look at each number one by one, then choose the ones that have both a whole number and a fraction." (SS, interview, item 2).

From the standpoint of the thinking process, SS already possesses relevant classification criteria and reported carrying out a thorough check. If the visual evidence confirms that the answer is correct, SS can be categorized as understanding the classification of mixed fractions. If not, the likely source of the problem lies in notation reading, carefulness, or the application of the criteria, rather than in the verbal definition itself. The article therefore needs to draw a clear distinction between "the student knows the attribute" and "the student can apply the attribute accurately."

SR selected only 2 1/4, even though 5 2/3 is also a mixed fraction. This initial answer shows that SR did not apply the classification criteria to all the options provided.

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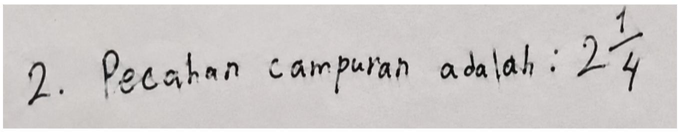


Figure 6. SR's answer on the FCUT, item 2.

"I check whether there is a whole number in front of the fraction." (SR, interview, item 2).
"I didn't pay attention to that number. I just chose the first one I saw." (SR, interview, item 2).
"Yes, it should be included as a mixed fraction." (SR, interview, item 2).

The interview shows that SR actually had prior knowledge of the attributes of mixed fractions. The failure occurred because the student did not scan and verify the entire list. Once attention was directed to 5 2/3, SR was able to recognize that this form also had a whole number and a fraction. SR's error is therefore not strong enough to be called a misconception about the definition of mixed fractions. The data more strongly support an interpretation of fragile understanding and an incomplete checking strategy. Cross-subject comparison confirms that correct classification requires two components: mastery of the concept's criteria and systematic application of those criteria. ST demonstrated both components, SS demonstrated the correct criteria but required verification of the written result, and SR had the initial criteria but had not yet applied them consistently.

Indicator 3: Giving Examples and Non-Examples of Common Fractions

This indicator examines students' ability to use a definition to generate examples and reject non-examples. This task is more demanding than simply recognizing a form, because students must construct suitable objects themselves and distinguish them from objects that do not fit.ST gave 1/2, 2/5, and 3/4 as examples and 5, 7, and 10 as non-examples. This selection was consistent with the forms of common fractions and whole numbers in the context of the task.

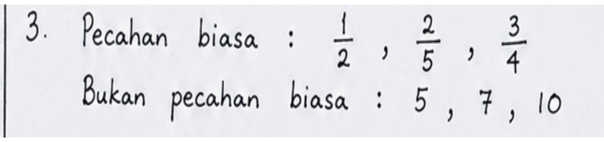


Figure 7. ST's answer on the FCUT, item 3.

"Because those numbers consist only of a numerator and a denominator." (ST, interview, item 3).
"Because those numbers are whole numbers, not fractions, like 5, 7, and 10." (ST, interview, item 3).

ST used the formal characteristics of a common fraction as the basis for constructing examples and non-examples. This response indicates that the definition of a common fraction had already functioned as a classification tool. However, the phrase "consists only of a numerator and a denominator" should be understood as an explanation appropriate for the elementary school level, not as a complete formal definition. In reporting the results, ST's explanation is sufficient to demonstrate mastery of this indicator because it is consistent with the objects selected.

SS selected 1/2, 2/3, and 2 as examples of common fractions, then grouped 1 1/3, 3 1/4, and 5/6 as non-examples. This response shows that SS could recognize some common fractions and mixed fractions, but made two important errors: including the whole number 2 as a common fraction, and excluding 5/6 from the group of common fractions.

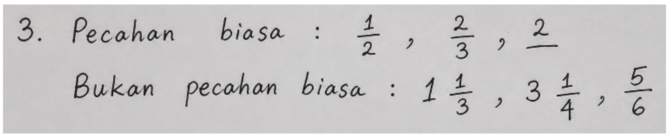


Figure 8. SS's answer on the FCUT, item 3.

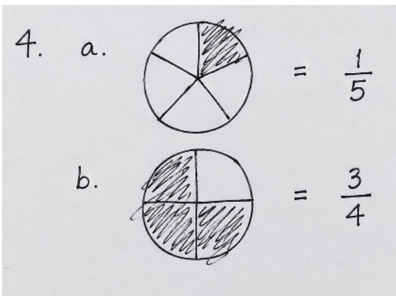
> *"I thought the number 2 was a common fraction because I think it can be written as a fraction." (SS, interview, item 3).*
>
> *"I got confused while grouping them, so I put 5/6 in the non-common-fraction group." (SS, interview, item 3).*
>
> *"If there is a numerator and a denominator, I consider it a fraction, but sometimes I still get confused telling them apart." (SS, interview, item 3).*

SS's explanation regarding the number 2 reveals interesting reasoning: mathematically, 2 can indeed be written as 2/1. However, in the context of a task asking for examples of "common fractions," students need to distinguish the whole-number form written as 2 from the common-fraction form written as a/b. SS's response therefore cannot be viewed entirely as incorrect understanding; the student demonstrated knowledge of equivalent representations but applied it outside the boundaries of the requested classification. Conversely, the error of placing 5/6 as a non-example shows that SS had not yet consistently applied the numerator–denominator criterion that had, in fact, already been stated. This contradiction reveals fragmented knowledge: SS could state the rule verbally, but the rule had not yet functioned as a stable basis when required to generate and classify examples. This is a strong indication that instruction needs to emphasize the comparison of examples, non-examples, and the reasoning behind their classification.

The written-answer data and interview excerpts for SR on this indicator are not yet fully available in the verified materials. A deep interpretation of SR cannot therefore be responsibly made at this stage. In the final manuscript, the analysis of SR needs to examine whether the student: recognizes the a/b form; distinguishes whole numbers, common fractions, and mixed fractions; and provides consistent reasoning for the groupings made. The absence of complete data is not a reason to fill this section with assumptions based on the low-ability category. In a reputable article, transparency about the limits of the data is stronger than making generalizations unsupported by primary evidence. On this indicator, ST showed stable use of the definition; SS showed partial understanding and a fragmentation between knowledge of equivalent representations and the demands of classification; while SR's profile must be determined once the complete primary data have been analyzed.

Indicator 4: Presenting Fractions in Mathematical Representations

This indicator assesses the ability to connect fraction symbols with part–whole representations. An accurate visual representation requires students to divide a whole into equal parts according to the denominator, then select a number of parts according to the numerator.



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ST drew 1/5 with five equal parts and one part shaded. ST also drew 3/4 with four equal parts and three parts shaded. Both answers show that ST used the denominator and numerator functionally in constructing the representation.

Figure 9. ST's answer on the FCUT, item 4.

"I divide the circle into several equal parts according to the denominator." (ST, interview, item 4).

"The denominator shows the total number of parts, while the numerator shows the parts that are shaded." (ST, interview, item 4).

The data interpretation shows that ST understood the two fundamental requirements of the area model: the parts must be equal in size, and the number of parts must match the denominator. In addition, ST correctly distinguished the roles of the numerator and the denominator. The quality of this response reflects coordination between symbolic and visual representation, rather than merely the ability to draw or shade.

SS divided the circle into the number of parts matching the denominator, but shaded a number of parts that did not match the numerator. For 1/5, SS shaded two parts; for 3/4, SS shaded only one part. Because the denominator was used correctly, SS's error was concentrated in coordinating the numerator with the number of parts selected.

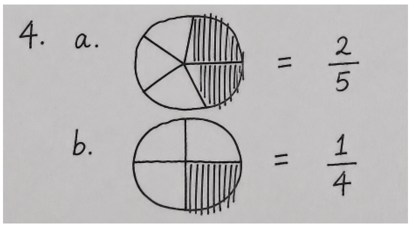

Figure 10. SS's answer on the FCUT, item 4.

"I draw the circle first, then divide it according to the denominator." (SS, interview, item 4).

"The number at the bottom shows the number of parts, while the number at the top shows the shaded part." (SS, interview, item 4).

"I was a little confused when dividing the circle, but I tried to divide it according to the numbers in the fraction." (SS, interview, item 4).

The gap between what SS said and what SS drew is the most important finding in SS's data. SS could state the rule declaratively, yet the resulting visual representation actually reversed, or failed to apply, the numerator rule. This indicates that the student's knowledge was not yet operationally integrated. In other words, the student "knew" the roles of the numerator and denominator when asked, but had not always "used" that knowledge to control their actions while completing the task. The statement that the student felt confused while dividing the circle also needs to be interpreted cautiously. The difficulty could involve dividing the circle into equal parts, yet the answer evidence shows that the number of parts already matched the denominator. The main problem is therefore more likely located in monitoring the number of shaded parts, or in attention to the numerator. In the discussion, this finding may point to the need for activities that require students to match symbols, area models, and verbal explanations simultaneously.

SR's visual representation data are not yet fully available in the verified materials. Before any interpretation is made, the researcher needs to examine whether SR is able to: divide a circle into equal parts, determine the number of parts according to the denominator, and shade according to the numerator. Interviews need to be used to distinguish technical drawing difficulty from difficulty in understanding the structure of fractions. Comparison on this indicator shows that ST coordinated symbols and the area model in an integrated way, SS had verbal knowledge but had not yet shown stability in visual application, while SR's data must be completed so that cross-subject conclusions remain grounded in evidence.

Indicator 5: Applying the Concept in Problem Solving

This indicator assesses students' ability to interpret information in a contextual problem, determine the sequence of fraction operations, carry out the calculation, and make sense of the result. The flour problem requires two relations: determining the remainder through subtraction, and determining the number of bags through division.

ST converted 2 1/2 to 5/2, equated the denominators, calculated the remaining flour as 5/4 kg, then divided 5/4 by 1/4 to obtain five bags. ST recorded the known and asked-for information and wrote out the solution steps in sequence.

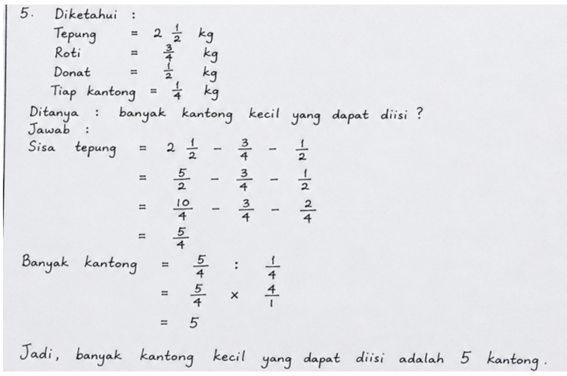
5. Diketahui :
Tepung = $2\frac{1}{2}$ kg
Roti = $\frac{3}{4}$ kg
Donat = $\frac{1}{2}$ kg
Tiap kantong = $\frac{1}{4}$ kg
Ditanya : banyak kantong kecil yang dapat diisi ?
Jawab :
Sisa tepung $= 2\frac{1}{2} - \frac{3}{4} - \frac{1}{2}$
$= \frac{5}{2} - \frac{3}{4} - \frac{1}{2}$
$= \frac{10}{4} - \frac{3}{4} - \frac{2}{4}$
$= \frac{5}{4}$
Banyak kantong $= \frac{5}{4} : \frac{1}{4}$
$= \frac{5}{4} \times \frac{4}{1}$
$= 5$
Jadi, banyak kantong kecil yang dapat diisi adalah 5 kantong.

Figure 11. ST's answer on the FCUT, item 5.

> *"First, I find the remaining flour by subtracting the flour that was used from the initial amount. After getting the remainder of 5/4, I divide it by 1/4." (ST, interview, item 5).*
>
> *"Because I have to find the remaining flour first before determining how many bags can be filled." (ST, interview, item 5).*
>
> *"I recalculated all the steps and made sure the fraction operations were correct. After checking, the result was still 5 bags." (ST, interview, item 5).*

ST's explanation shows an understanding of the problem's semantic structure, not merely the ability to execute an algorithm. ST understood that the word "remainder" points toward subtraction, and that the question about the number of bags points toward dividing the remainder by the capacity of each bag. The re-checking demonstrates control over the solution process. ST therefore showed an application of the fraction concept that was systematic, meaningful, and reflective.

SS also recognized the need to subtract the flour used and to divide the remaining flour by the capacity of each bag. SS arrived at the answer of five bags, but some steps were not written out completely. The interview revealed that SS carried out part of the process mentally and did not re-check the entire procedure.

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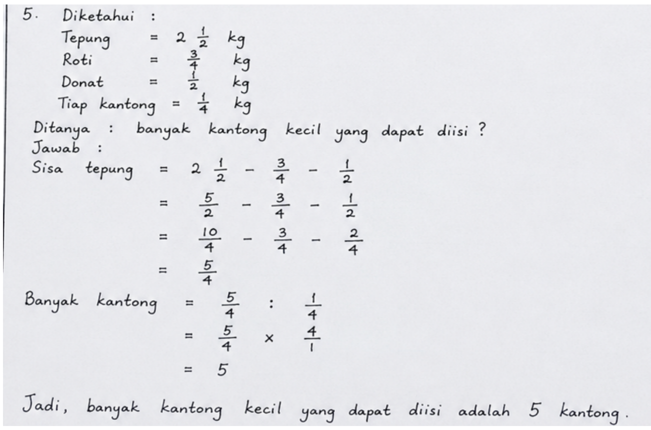

Figure 12. SS's answer on the FCUT, item 5.

"I first subtracted the flour used for the bread. After that, I calculated the flour used for the donuts again, but I forgot to write down that calculation. After getting the remainder, I divided it by the amount in each bag." (SS, interview, item 5).

"Because the remaining flour has to be divided into several small bags." (SS, interview, item 5).

"I only looked at the final result and didn't recalculate all the steps." (SS, interview, item 5).

SS's data show that the student understood the overall flow of the problem and the meaning of division as the process of distributing the remaining flour into fixed-size bags. This understanding goes beyond simply choosing an operation at random. However, "forgetting to write down the calculation" and not re-checking the work indicate that the solution process was not consistently documented or monitored. This weakness can increase the risk of error when the numbers or the structure of the problem become more complex.

SS can therefore be understood as having functional understanding of the problem situation, but not yet showing strong procedural regulation. In reporting the results, it is important to distinguish between students who arrive at a correct answer through a structured process and students who arrive at a correct answer but cannot show a complete trace of their procedure and checking.

The answer and interview data for SR on the contextual problem are not yet fully available in the verified materials. The final analysis needs to examine whether SR understands the word "remainder," can choose subtraction before division, is able to perform fraction operations, and can explain the meaning of the result in terms of the number of bags. If SR makes an error, the interview needs to reveal whether the error stems from misinterpreting the context, choosing the wrong operation, or a procedural difficulty in computing fractions. Comparatively, ST showed integrated conceptual understanding and process control, SS showed understanding of the problem structure but limited checking and documentation of the procedure, while the interpretation of SR must be resolved using complete primary data.

Overall, the results show that understanding fractions is not a single ability. ST demonstrated integration among definition, procedure, representation, reasoning, and answer-checking. SS possessed a number of correct declarative pieces of knowledge, but this knowledge was not always connected to mathematical action, particularly in the tasks involving representation and example/non-example classification. SR showed initial knowledge of some features of fraction forms, but had not yet constructed the value relation and had not yet applied the criteria consistently. These findings underscore the added value of task-based interviews. Written answers alone can show whether an answer is correct or incorrect, but interviews reveal whether a response was formed through relational understanding, the application of an as-yet-unstable rule, carelessness, or a mistaken conceptual belief. In the article, this synthesis can serve as a bridge from the Results section to the Discussion without repeating the entire data account.

Discussion

This study shows that fifth-grade students' understanding of fraction concepts cannot be assessed based solely on the accuracy of final answers or on mathematics ability categories derived from report card scores. Analysis of written answers, students' work images, and task-based interviews reveals three main patterns. First, there are students who can consistently connect definitions, procedures, representations, mathematical reasoning, and answer-checking. Second, there are students who can state fraction definitions or rules but have not yet applied them stably in classification and visual representation tasks. Third, there are students who still rely on surface features, such as visual similarity or the presence of certain numbers, without fully understanding the quantitative relationship between the numerator, the denominator, and the whole. These findings show that understanding fractions is multidimensional. Understanding encompasses not only the ability to perform operations, but also the ability to interpret a fraction as a single value, understand the part–whole relationship, connect various forms of representation, classify number forms, choose an operation based on contextual meaning, and evaluate the reasonableness of an answer (Copur-gencturk, 2021; Copur-Gencturk & Doleck, 2021; MacDonald et al., 2025; Van Hoof et al., 2020). Correct answers therefore need to be interpreted together with students' reasoning and the representations used, while incorrect answers need to be examined to distinguish procedural errors, carelessness, partial understanding, and indications of misconception.

Integrated Conceptual Understanding

One pattern of findings shows a relatively integrated understanding of the fraction concept. Students with this profile could determine equivalent fractions, explain that equivalent fractions have the same value even though their numerators and denominators differ, and explain the procedure of multiplying by the same number. These students could also identify mixed fractions based on the presence of a whole number and a common fraction, distinguish common fractions from whole numbers, and represent 1/5 and 3/4 using equal parts of a circle. On the contextual problem about flour, these students understood that the first step is to find the remainder through subtraction, then determine the number of bags by dividing the remainder by the capacity of each bag.

This pattern supports Skemp's distinction between instrumental and relational understanding. Instrumental understanding relates to the ability to execute a rule, while relational understanding includes understanding the relationships and the reasons for using a rule. In these findings, the fraction procedures did not stand alone (Kafetzopoulos & Psycharis, 2022; Vorhölter, 2018). The subtraction operation was interpreted as a way of determining the remaining flour, while division was interpreted as a way of determining the number of 1/4-kg groups. The procedures used therefore had a semantic and conceptual basis, rather than being merely a memorized sequence of steps.

These findings are consistent with research on fraction knowledge, which states that conceptual knowledge encompasses the concepts and principles underlying a domain, while procedural knowledge encompasses the sequence of actions used to solve a problem (Kaiser et al., 2025; Liljekvist et al., 2017) . The relationship between the two is mutually supportive. In the development of fraction learning, conceptual understanding can help students use

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procedures more flexibly, particularly when they encounter task forms or contexts that differ from routine practice.

Nevertheless, the ability to determine equivalent fractions by multiplying the numerator and denominator by the same number does not by itself indicate the deepest level of relational understanding. A stronger explanation would emerge if students could prove the equivalence of 2/3, 4/6, and 6/9 using an area model or a number line. The results of this study are therefore more accurately described as showing relatively strong operational conceptual understanding—students could use and explain the procedure correctly, but the depth of relational justification can still be explored further.

The Gap Between Verbal Knowledge and Representation

The most crucial finding in this study is the gap between the knowledge students could express verbally and its application in mathematical action. Some students could state that the denominator shows the total number of parts and the numerator shows the number of shaded parts. However, when asked to draw 1/5, students divided the circle into five parts but shaded two. In the task of drawing 3/4, students divided the circle into four parts but shaded only one. This gap shows that declarative knowledge had not yet fully become operational knowledge. Students could state the correct rule, but had not yet used that rule as a basis for controlling their actions when constructing a visual representation. This explains why oral assessment alone can give an overly optimistic picture of students' understanding, while written assessment alone can overlook prior knowledge that students actually possess. Combining students' work images with interviews allows for a more accurate identification of which parts of the concept have been understood and which have not yet been integrated.

These findings are consistent with research on fraction visualization reporting that students' difficulties often arise when dividing a whole into equal parts, keeping the whole unit consistent, or coordinating the numerator and denominator in a drawing. Errors of this kind show that students have not yet fully interpreted a fraction as a part–whole relationship, but instead still view it as two numbers with separate roles (Čadež & Kolar, 2018; MacDonald et al., 2025). When the numerator and denominator are not coordinated, representational errors can develop into difficulties with equivalent fractions and fraction operations. Within Bruner's framework, conceptual understanding can develop through enactive, iconic, and symbolic representation. In learning fractions, students need to connect the activity of dividing concrete objects, part–whole pictures, and the a/b symbol. When students can explain the function of the numerator and denominator but cannot draw them accurately, the connection between symbolic or verbal representation and iconic representation has not yet been stably formed. Instruction therefore cannot rely on explaining the rule alone, but must provide repeated experience in translating one concept into various forms of representation.

Classification and Concept Boundaries

The next finding shows that the ability to classify fraction forms needs to be understood as the ability to apply the concept's characteristics consistently, not merely to recognize one familiar example. In the task of identifying mixed fractions, a student could explain that a mixed fraction has a whole number and a common fraction. However, the student selected only one example and overlooked another example with the same feature. After being asked to look again, the student realized that the other form was also a mixed fraction. This pattern shows that the student had prior knowledge of the features of mixed fractions but had not yet applied it through a systematic checking procedure (Lee et al., 2017). The error is more likely related to selective attention and insufficient verification of all the options than to a complete lack of knowledge of the definition. A response of this kind should therefore not immediately be labeled a misconception. A more cautious interpretation is that the student had partial understanding and an incomplete classification strategy. A misconception can only be claimed when an incorrect understanding is consistently maintained across several tasks or after probing questions have been given.

A classification difficulty also arose when a student included the whole number 2 as a common fraction, reasoning that 2 can be written as 2/1. This reasoning has a mathematical basis, since any whole number can indeed be expressed as a fraction with a denominator of one. However, in the context of a task asking for examples of common fractions based on their written form, the number 2 written as a whole number should not be classified as a common fraction. This student had not yet clearly distinguished between the value of a number, the form in which it is represented, and the concept category required by the task. In addition, classifying 5/6 as a non-example of a common fraction shows an inconsistency between the definition a student could state and the application of that definition during classification. This finding can be interpreted as fragmented knowledge: the student possessed correct pieces of information—for example, that a fraction has a numerator and a denominator—but that information had not yet functioned as a consistent conceptual structure. Research on fraction learning also reports that students often process the numerator and denominator separately, or use surface features to make decisions, making them more prone to errors in classification, comparison, and operations.

Interpreting Context and Verification

On the contextual problem about flour, a difference was found between students who used a structured procedure and students who used a relevant strategy but had not carried out thorough verification. Students with stronger understanding identified the known and asked-for information, found the remaining flour through subtraction, divided that remainder by the capacity of each bag, and recalculated the solution steps. Other students also chose subtraction and division appropriately and arrived at the correct answer, but some steps were not written down and the answer was not re-checked. This finding shows that fraction problem solving does not depend solely on fluency in computation (Hu et al., 2025; Rich et al., 2019; Wilkie & Hopkins, 2024). Students need to build a model of the situation: the word "used" indicates subtraction from the initial amount; the word "remainder" refers to the result of that subtraction; and the information that each bag holds 1/4 kg indicates the need to divide the remainder into fixed-size groups. Students who understand these semantic relationships can explain why subtraction is performed before division. In contrast, students who merely memorize the order of operations risk making errors when the context or the order of the information is changed.

The lack of re-checking shows that success in reaching the final answer is not always accompanied by strong process regulation. Verification functions not only to catch computational errors but also to assess whether a result makes sense in relation to the context. In this problem, students could be asked to check whether five bags is a reasonable result when the remaining flour is 5/4 kg and each bag holds 1/4 kg. Questions of this kind encourage students to connect a procedural result with its quantitative meaning.

Comparison with Previous Research

Overall, the findings of this study are consistent with research that positions fraction understanding as a combination of conceptual and procedural knowledge. Research on elementary school students distinguishes conceptual knowledge of fractions in terms of the ability to compare, apply, and visualize fractions, while procedural knowledge is reflected in

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the ability to convert, simplify, and perform operations. In this study, stronger understanding appeared when students were able to visualize fractions, explain the equivalence of values, and apply operations based on context; conversely, difficulty appeared when verbal knowledge was not connected to representation or classification. Consistency is also evident with systematic reviews of fraction instruction that emphasize the need to connect real-world situations, manipulatives, pictures, spoken language, and written symbols. (Lee & Lee, 2021; Purnomo et al., 2022). The data in this study show that the problem does not lie solely in students' ability to use a single representation. Students can succeed with one representation—for example, stating a definition—yet fail when translating the concept into another representation, such as a circle diagram or a word problem. Instruction therefore needs to explicitly build bridges between representations rather than presenting them separately.

The finding regarding the use of surface features is also consistent with research reporting that some elementary school students view fractions as meaningless symbols or as always smaller than one. When students have not yet understood a fraction as a number and a quantitative relationship, the numerator and denominator are more easily treated as two independent whole numbers. This pattern can explain why students can state part of a rule but cannot yet apply that same rule consistently in classification, representation, or problem solving. At the same time, the findings of this study complement previous research by showing the importance of examining the consistency among four forms of evidence: symbolic answers, students' work images, verbal explanations, and strategies used on contextual problems. Unlike studies that measure only test outcomes, this study shows that a correct answer can be reached through understanding of differing quality. One student may choose the correct operation but not check the procedure, while another may explain the concept but fail to represent it. Assessment of fraction understanding therefore needs to use multiple forms of evidence so that the diagnosis of students' difficulties is more accurate.

Instructional Implications

These findings point to several instructional implications. First, fractions need to be taught as values and relationships, not merely as a pair of a numerator and a denominator that can be manipulated using certain rules. Students need to understand that a/b represents a parts out of b equal parts, and that a fraction is also a number that can be compared, placed on a number line, and used in real situations. Second, instruction needs to use a sequence of concrete objects, pictures, and symbols that are interconnected. Students can divide paper or concrete objects into equal parts, draw the result of that division, write the fraction, and explain its meaning. For equivalent fractions, students need to see that further subdividing the same parts does not change the quantity—for example, 2/3 can be re-represented as 4/6 and 6/9.

Third, teachers need to use classification tasks that include examples, non-examples, and boundary cases. Students can be asked to distinguish between 2, 2/1, 5/6, 2 1/4, and 9/4, and then explain the reasoning behind their groupings. This activity helps students distinguish between the value of a number, its form of representation, and the category of fraction type. Fourth, contextual problems need to be accompanied by questions that encourage justification and verification, such as "Why did you use subtraction?", "What does the result of the division mean in the story?", "How can a picture or a number line prove your answer?", and "Does your answer make sense?" Questions of this kind make students' thinking processes visible and can be used by teachers to provide diagnostic feedback.

Conclusion

Overall, this study shows that understanding of the fraction concept develops through the interconnection of definition, representation, procedure, reasoning, and verification. The difficulties identified were primarily a failure to connect existing knowledge into consistent mathematical action. Instruction and assessment of fractions therefore need to give students room to explain, draw, classify, use context, and re-check their answers. This approach can help teachers identify whether a student has understood fractions conceptually, has only mastered part of the knowledge, or still requires concrete and representational support. The interpretation of these results needs to take the study's limitations into account. This study involved three students from a single school and focused on five indicators of fraction concept understanding. The findings therefore provide an in-depth picture of the cases studied, not a statistical generalization to all elementary school students. The mathematics ability category was used to select varied cases, not to assign a permanent label to students' ability. In addition, the data were obtained through written answers and interviews. Interviews depend on students' ability to articulate their thinking, so some mental processes may not have been fully expressed. Claims regarding misconceptions must also be limited to responses consistently supported by both written answers and verbal explanations; a single incorrect answer is not sufficient to establish that a student holds a persistent misconception.

Conflict of Interest

The authors declare no conflict of interest.

Auhor Contributions

S.P.O. conceptualized the research idea presented and collected the data. The other three authors (M.I., T. and P.) actively contributed to the development of the theory, methodology, data organization and analysis, discussion of the results, and approval of the final version of the work. All authors confirm that they have read and approved the final version of this manuscript. The percentage contributions for the conceptualization, drafting, and revision of this paper are as follows: S.P.O.: 40%, M.I.: 20%, T.: 20%, and P.: 20%.

Data Availability Statement

The authors state that the data supporting the findings of this study will be made available by the corresponding author, S.P.O., upon reasonable request.

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